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The use and calibration of distance measuring equipment for precise mensuration of dams (Revised)

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This report is intended as a practical guide to land surveyors who use distance measuring equipment (DME) for making precise measurements of movements in dams and other large structures. The report consists of two parts: First, DME calibration, error sources, atmospheric corrections and correct use; Second, special techniques of trilateration and data reduction for precise mensuration on dams. This revised report contains several new sections dealing with error estimation, adjustment, and refractive index corrections.

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PREFACE This report is a revised version of a report by the same title, report number ETL–0048, dated March 1976. This revised report contains several new sections dealing with error estimation, adjustment, and refractive index corrections.

COL Daniel L. Lycan, CE, was Commander and Director of ETL during the report preparation. Mr. Robert P. Macchia was the Technical Director.

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INTRODUCTION

The Corps of Engineers needs precise measurement of displacements within large engineering structures such as locks and dams. The regulations state that "Civil Works structures whose failure or partial failure would endanger the lives of the public or cause substantial property damage will be continuously evaluated to insure their structural safety and stability, and operational adequacy. Such evaluations, based upon periodic inspections supported when appropriate by programs of instrumentation, will be conducted to detect conditions of significant structural distress or operational inadequacy and to provide a basis for timely initiation or restorative and remedial measures."[1]

One of the most effective programs of instrumentation is high precision survey. This method provides a direct measure of displacement as a function of time, is reliable, and has fewer problems of evaluation than most other types of instrumentation. It is the purpose of this manual to provide assistance to the surveyor faced with the problem of making these precise surveys in a timely and cost-effective manner. The manual addresses itself to one particular method of precise survey, trilateration, and is not intended to provide instruction in general surveying, although some of the techniques discussed do apply.

Without question the most important recent advance in surveying has been the introduction of modern optical distance measuring instruments. In order to meet the needs of the surveyor these instruments come in a bewildering variety in terms of cost, range, accuracy, and ease of operation. When properly calibrated and used, distance measuring equipment (DME) is much more than simply a replacement for the steel tape. Instead, it provides new techniques, which may as yet be unfamiliar to the surveyor, provides substantial savings in time and manpower, and leads to higher accuracies.

Unfortunately, DME also provides new problems, which must be dealt with. For example, the instruments must be calibrated together with their reflectors, error sources must be examined, and the proper use of temperature and pressure monitoring equipment becomes essential. A further purpose of this manual then is to deal with the problem of instrument selection, calibration, and use, particularly where measurements of high accuracy are required.

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[1] US Army Corps of Engineers, "*Periodic Inspection and Continuing Evaluation of Completed Civil Works Structures*," Regulation No. 1110-1-100, 26 Feb 73.

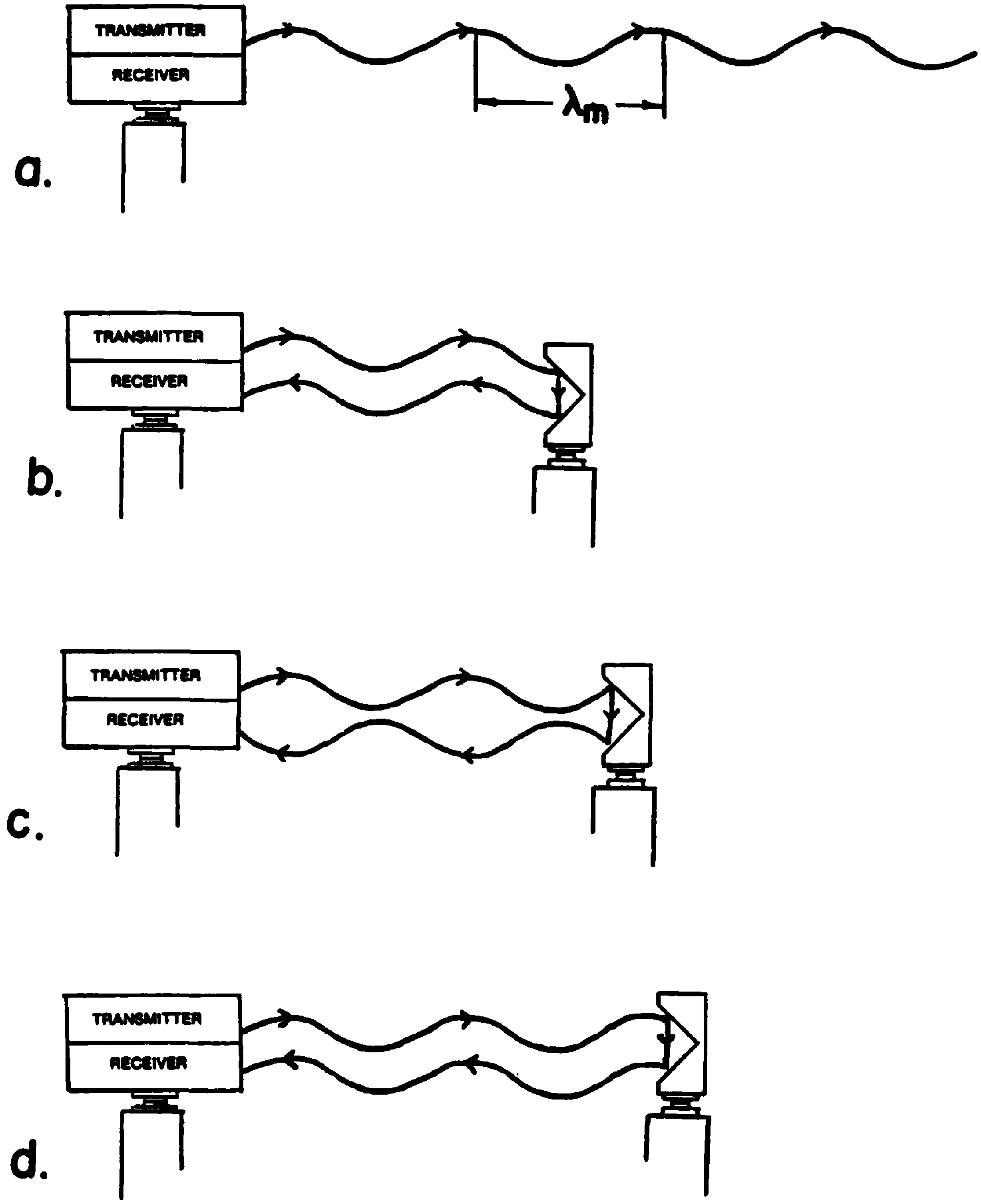


Figure 1. Phase Comparison.

THE CALIBRATION AND USE OF DISTANCE MEASURING EQUIPMENT

DME OPERATION. All modern DME measure distance by timing, in an indirect fashion, how long it takes light to make a round trip to a reflector. By knowing the velocity of light, the distance may be calculated from $2d = vt$, where d is the distance to the reflector, v is the velocity of light, and t is the time required for light travel to the reflector and back. Light travels 1 foot in approximately a nanosecond (1×10^{-9} seconds). If it were possible to time directly a pulse of light as it traveled to a target and back, the number of nanoseconds divided by 2 would be roughly the number of feet to the target. This is the principle of the laser range finder used by the Army. For the purpose of artillery, an accuracy of a few feet is sufficient, and direct timing of pulses provides the desired result. However for surveying, where accuracy requirements are much greater, another technique must be used to avoid the problems of timing a pulse to a small fraction of a nanosecond. In surveying a continuously operating source of light is used, and this light is modulated in a known way. In the case of DME, modulation simply means that the light is turned off and on in a regular fashion, usually sinusoidally. In figure 1a, a DME is shown emitting a modulated beam of light. The sinewave represents the amplitude of the light along the path of the beam at an instant of time. The modulation wavelength λ_m, not to be confused with the natural wavelength of the light itself, is determined by the rate at which the light is modulated and by the velocity at which it is traveling; $\lambda_m = v/f$, where λ_m is the modulation wavelength, v is the velocity at which the light is traveling, and f is the frequency at which the light is being modulated. In figure 1b, the light has been reflected from a mirror or corner cube reflector and has been returned to the instrument. A measurement is then made of the phase difference between the light proceeding toward the reflector and that returning. In figure 1b, the reflector has been placed so that it is exactly a whole number of modulation wavelengths away from the instrument. The returning light is then found to be in phase with the outgoing light, and the operator knows that the distance to the reflector is a whole number, n, times the modulation wavelength divided by two ($n\lambda_m/2$). In figure 1c, the relector has been moved one-quarter wavelength farther away from the instrument. The returning beam is now one-half wavelength or 180° out of phase with the outgoing beam. The operator knows the distance to the reflector is now a whole number times the modulation wavelength divided by two, plus one-quarter of a modulation wavelength ($n\lambda_m/2 + \lambda_m/4$). Finally in figure 1d, the reflector has again been moved one-quarter wavelength farther away, and again the beam returns in phase with the outgoing beam. Note that in moving the reflector by one-half wavelength, the phase change has been a full wavelength or 360°. This also means that the same answer will be obtained every one-half wavelength so that additional modulation wavelengths are required to resolve ambiguities. In an instrument was built with a 20-meter-modulation wavelength, the same result would be obtained every 10 meters. For example, if an answer of 3.462 meters resulted from a measurement with the 20-meter-modulation wavelength, the operator would not know if the complete answer would be 3.462 meters, 13.462 meters, 23.462 methers, or perhaps 2183.462 meters. It would then be necessary to switch to a longer modulation wavelength. If the new wavelength were 10 times longer, the instrument would be able to determine the correct value of the figure in the tens place. To determine the figure in the hundreds place, the wavelength would again be increased ten times. This procedure would be repeated until the entire distance was resolved. Some instruments require five or six modulation frequencies to resolve ambiguities completely. By using the phase comparison method, instruments can be built that measure to better than one-thousandth of a modulation wavelength. Thus, certain instruments have resolutions finer than 0.001 meter.

This in brief is how the modern DME works. It requires a knowledge of the velocity of light and uses modulation of the light at a precisely known frequency, and phase comparison to obtain the distance. Each of these factors, however, introduces an error into the measurement. The next section deals with some of these errors.

DME ERROR SOURCES. This section refers specifically to error sources, which are an inherent property of the instrument itself and are in addition to external factors, such as plumbing and measurements of refractive index. Most manufacturers of DME list the error of their instruments as (a) mm + (b) mm/km, where (a) and (b) are maximum values for a particular instrument model and manufacturer.

In the (a) portion of the error, several small errors are lumped together, which are independent of the length of the line being measured. The more important of these are

1. Instrument resolution.
2. Cyclic or delay line error.
3. Instrument-reflector calibration.
4. Offset.
5. Pointing Error.

The (b) portion of the error is due to the short- and long-term variations in the frequency standard used to control the modulation frequency. This is usually a quartz crystal, which may or may not be mounted within a small oven for temperature control.

While the errors described in (a) are generally grouped together, it is helpful to examine them individually, as occasionally special techniques may be used to lower the magnitude of certain of them.

1. Instrument Resolution. Resolution is a property of the instrument that results from its original design. In the case of DME, it might be defined as the smallest change in distance to a target that causes a corresponding change in the reading

obtained from the instrument. The resolution can be no finer than the scale or digital display can be read. On the other hand, a display that can be read to a millimeter is no assurance that the resolution of the instrument is also a millimeter.

The simplest test for resolution is to mount a reflector so that it may be moved back and forth along a line connecting the reflector and the DME. Movement of a half meter or so is sufficient. Affix a scale next to the reflector so that the distance the reflector is to be moved may be accurately measured. Make measurements with the reflector at several different positions along the scale. The calculation of resolution may then be performed as shown in table 1.

Table 1. DME Resolution Test

Reflector Position (meters) (1)	DME Measurement (meters) (2)	Difference (meters) (3)
0.040	132.560	132.520
0.151	132.668	132.517
0.228	132.749	132.521
0.400	132.916	132.516
0.506	133.019	132.513
0.591	133.110	132.519
	Mean =	132.5177

Standard deviation, σ (resolution) = 0.003 meters.

In this example, the DME was positioned some 132 meters from the scale and reflector. The reflector was positioned at 0.040, 0.151, 0.228, 0.400, 0.506 and 0.591 meters according to the scale (column 1 of the table). A measurement of the distance was made with the DME for each position of the reflector (column 2). The values in column 1 are then subtracted from the corresponding values in column 2 to obtain the differences (column 3). Finally, the standard deviation is taken of the values in Column 3, and the result, 0.003 meter, is the approximate resolution of the instrument.

2. Cyclic or Delay Line Error. If the manner in which the light is modulated distorts the sinusoidal pattern of the outgoing beam or if the phase comparison technique of measuring the returning beam is less than perfect, a cyclic error will occur.

The error is named cyclic because it repreats itself every modulation wavelength. If the effective modulation wavelength of a particular instrument is 10 meters and the cyclic error for a measurement of 6 meters is 4mm, then the cyclic error at 16, 26, and 36 meters would also be 4mm. At the same time, the instrument might have zero cyclic error at 1, 11, and 21 meters. A determination of cyclic error consists of making comparative measurements throughout a modulation wavelength. (If a full modulation wavelength is too long for convenient use, measure through one-half a wavelength and consider the cyclic errors in the second half to be symmetrical with those in the first half.) From the manufacturers specifications, choose the highest modulation frequency (this will usually be between 10 and 80 megahertz). The effective modulation wavelength will be $\lambda_m = 3 \times 10^8/2f$ meters, where λ_m is the modulation wavelength, and f is the highest modulation frequency. For an instrument operating at a frequency of 15 megahertz, the effective modulation wavelength would be $\lambda_m = 3 \times 10^8/2(15 \times 10^6) = 10$ meters. Divide this distance into at least 10 equal parts. For the 10-meter wavelength, a convenient value might be 13 increments of 0.8 meter each.

Make measurements with the DME of the distance to a reflector as it is moved in increments of length along a straight line over an entire modulation wavelength. Record the data in a manner similar to that shown in table 2. The measurements (table 2) contain cyclic or delay line errors, but also include resolution errors as found in the previous section.

Table 2. Cyclic or Delay Line Error

Reflector Position (meters)	DME Measurement (meters)	Difference (meters)
0	80.503	80.503
0.8	81.305	80.505
1.6	82.111	80.511
2.4	82.909	80.509
3.2	83.710	80.510
4.0	84.506	80.506
4.8	85.306	80.506
5.6	86.102	80.502
6.4	86.900	80.500
7.2	87.698	80.498
8.0	88.496	80.496
8.8	89.297	80.497
9.6	90.101	80.501
10.4	90.904	80.504
	Mean =	80.5034

Standard deviation $\sigma = 4.8$ mm

When dealing with errors of this type, the simple difference between the two cannot be taken. If x and y are errors of different types, the sum is given by $z^2 = x^2 + y^2$. In table 2, the cyclic error plus the resolution error was 4.8 mm. The cyclic error alone would be $y^2 = (4.8)^2 - (3.0)^2$; $y = 3.7$ mm.

If cyclic errors are large, a calibration curve may be drawn up and corrections made to the distances read from the instrument. Some manufacturers will supply a calibration curve upon request.

3. Instrument-Reflector Calibration. When a DME is received from the manufacturer, one or more reflectors are usually received at the same time, and these have been assigned a calibration constant by the manufacturer. This constant is to be added or subtracted from a distance reading in order to obtain a correct distance. In most cases, the constant is sufficiently accurate for routine work. However, for greater precision or for reflectors that are obtained from other sources, it is necessary to determine accurately the constant of each reflector. Figure 2 shows two reflectors, both

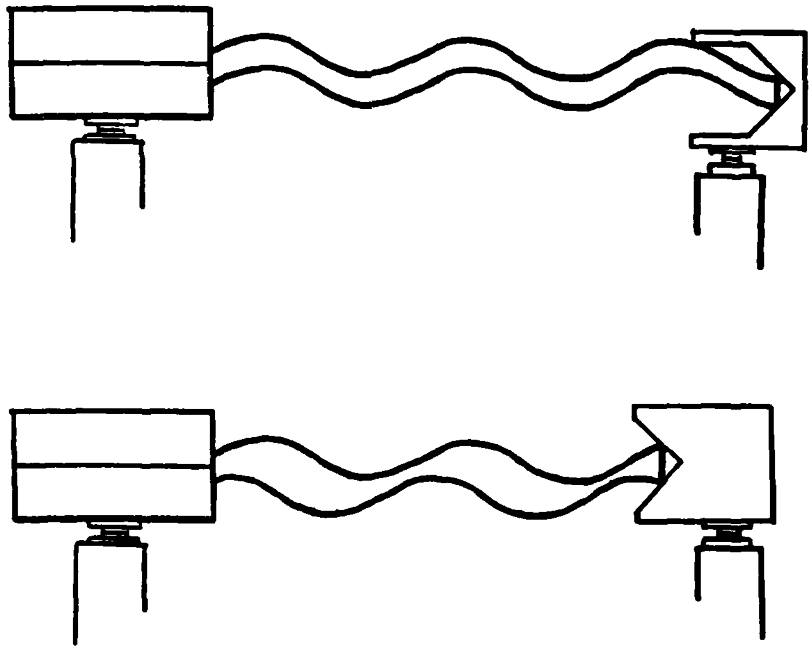


Figure 2. Reflector Calibration.

of which are mounted at the same distance from the measuring instrument. Because the reflectors are at different positions within their mounts, they give different readings for the same distance. The difference between the true distance and the reading obtained with a particular reflector is the instrument-reflector constant. The constant may be determined in two ways. The simplest method is to measure several short lines with accurately known lengths. The difference between the measured length and known length is the instrument-reflector constant. The lines should be shorter than 300 meters in length and the measurements must be corrected for temperature and pressure (See the section on Atmospheric Corrections). At least three different lengths should be measured to sample the modulation wavelength. If only one length is available, two additional points may be established 5 meters on either side of the original end point and precisely along the line of sight between the end points of the original line.

If a baseline is not available, the instrument-reflector constant may be determined by following these steps. Lay off three points along a straight line. Mark the second point about 100 meters from the first. Choose a distance between the second and third points of about 75 meters. Set the DME on the first point, and measure to the reflector plumbed over the second point. Next, measure to the *same* reflector plumbed above the third point. Both measurements will be in error by the amount of the reflector constant, but the difference of the two measurements will be the true length, free of this error. Move the instrument to the second point, and measure the distance to the *same* reflector mounted above the third point. The amount that must be added or subtracted from this measurement to obtain the true length is the reflector constant. If measurements are completed within a period of 2 hours, temperature and pressure measurements need not be made. An example of the determination of a reflector constant is given in table 3. The reflector constant is only good for that particular instrument-reflector combination.

Table 3. Determination of Reflector Constant

LINE	LENGTH (from DME)
Point 1 to Point 3	183.409 (from DME)
Point 1 to Point 2	113.406 (from DME)
Difference	70.003 = true length
True length	70.003 = B (difference)
Point 2 to Point 3	70.036 = A (from DME)
Reflector Constant	-.033 = (B - A)

0.033 must be subtracted from the instrument reading to correct for the reflector constant.

	True Distance
Point 1 to Point 2	113.406 - .033 = 113.373
Point 1 to Point 3	183.409 - .033 = 183.376

If the reflector is used with another instrument, the constant will have to be redetermined for that instrument. Finally, the tests of reflector constant should be repeated several times with the difference from test to test being no larger than the resolution of the instrument.

If other reflectors are to be used with the DME, they may be calibrated in the same manner, or one reflector may be selected as a standard. When the standard reflector has been calibrated with the instrument, the reflector may be mounted at a distance of approximately 100 meters, and it may be used to measure the distance. After applying the reflector constant to the measured distance, the length is considered to be correct. Each of the other reflectors is then mounted in turn over the same point. Any difference in the measured distance is due to a difference in reflector constant. Table 4 gives an example of the calibration of several reflectors. The differences in reflector constants shown in the table can occur in actual practice, which shows the necessity for calibration of each reflector.

Table 4. Calibration of Several Reflectors

Reflector	Measured Length	Constant
1	100.687	-0.031
2	100.680	-0.024
3	100.696	-0.040
4	100.582	+0.074
5	100.580	+0.076

Measurement of line with standard reflector	100.689
Reflector Constant	-.033
True line length	100.656

4. Offset. The reflector calibration includes two sources of error. The first error is caused by the reflector not being optically above the point over which it is plumbed. The second error is caused by the DME electrical center, the point from which the instrument measures, not being above the point over which it is plumbed. However, both errors are corrected when the instrument-reflector combination is calibrated. On the other hand, if the electrical center of the instrument should shift as the electronics age, the instrument-reflector constant would no longer compensate for this shift, and an offset error would result. Even so, it is possible to measure the magnitude of the offset error. At weekly intervals, simply measure a short line (100 meters) using the same reflector (the line should be outside so that the instrument will be subject to

a variety of temperatures). If each measurement is made using the same procedures, the differences in length in excess of the resolution error are due to changes in the offset or electrical center of the DME. Temperature and pressure corrections must be made to these measurements. Offset changes of 5 millimeters may occur in some instruments.

5. Pointing Error. The modulation wavefront issuing from a properly designed and operating instrument is at all points equidistant from the center of the instrument (figure 3). It might be likened to the waves around a stone dropped into water. Sometimes, however, the wavefront may be distorted in passing through the modulator, and then a portion of the wave may be ahead or behind the remainder. In figure 3, the instrument sees both reflectors as equidistant because the phase of the modulated wave is the same for both. If the instrument is moved in azimuth slightly, the distance that is read would change.

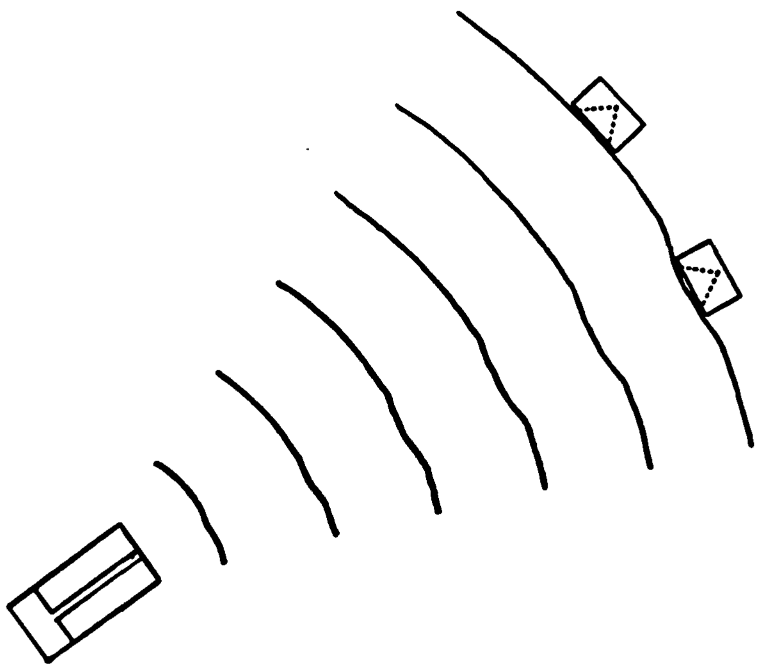


Figure 3. Pointing Error.

This type of error may be detected simply by multiple pointings at a reflector. If different pointings yield different answers, it may be necessary to take several readings in the field, swinging off the target and then back until two or three sets of readings agree well. Practice in the field may help eliminate this problem as an experienced operator tends to point and adjust an instrument in the same way for each measurement.

Some instruments have a signal strength or gain control. Also, the instrument will only operate when the signal strength lies between established maximum and minimum values. Always point the instrument, using the tangent screws, until a maximum signal strength is obtained, then use the gain control to obtain the proper signal level. If the tangent screws are used to obtain the proper signal level, the instrument may be mispointed and significant errors can occur. Always point for maximum return signal before making a measurement. A good measuring procedure is to point the instrument carefully, take five readings, then using the tangent screws, move the instrument off target. Repoint and take an additional five readings.

Total Error. The previous sections have treated individually the various types of error in the (a) portion of the error that may be present in DME. It may be, however, that the total (a) error is all that is desired. In this case, one series of measurements will provide an indication of the total error. A straight line of five monuments is required as shown below.

The instrument is set over each point in turn, and the following distances are measured:

From A: AB, AC, AE
From B: BA, BC
From C: CA, CB, CD, CE
From C: DC, DE
From E: EA, EC, ED

Use the same reflector for all measurements. If three prisms are required for longer lengths, one or two prisms may be masked off for the shorter measurements. Let AE be no longer than 750 meters and let no two segments be of the same length, or a multiple of the length, so that a good sampling of the modulation wavelength may be obtained. Apply refractive index corrections to the measurements, and attempt to measure all the lines during a single day. It is not necessary to know the true lengths of the segments. Calculate the total length, ℓ, of the line in the following ways:

$$\ell_1 = \frac{AB + BC + CD + DE + ED + DC + CB + BA}{2}$$

$$\ell_2 = \frac{AC + CE + EC + CA}{2}$$

$$\ell_3 = \frac{AE + EA}{2}$$

The error in each of the measurements consists of a random portion owing to noise, resolution, and cyclic error and of a constant portion owing to the offset and instrument-reflector calibration errors. The constant portion of the error is present with the same sign and approximately the same magnitude for each measurement. Thus, ℓ_1 contains this error $8/2 = 4$ times, ℓ_2 contains the error $4/2 = 2$ times, and ℓ_3, $2/2 = 1$ time. In table 5, the lengths measured on a five-station line (column 2), and the lengths ℓ_1, ℓ_2 and ℓ_3 are listed.

Table 5. Lengths of a Five-Station Baseline

Segment (1)	Measured Length (m) (2)	Length Corrected for offset (m) (3)
AB	81.236	81.232
AC	176.926	176.922
AE	690.149	690.145
BA	81.241	81.237
BC	95.696	95.692
CA	176.929	176.925
CB	95.692	95.688
CD	274.815	274.811
CE	513.222	513.218
DC	274.816	274.812
DE	238.409	238.405
EA	690.146	690.142
EC	513.226	513.222
ED	238.414	238.410

$$\begin{aligned} \ell_1 &= 690.1595 & \ell_6 &= 690.140 \\ \ell_2 &= 690.1515 & \ell_7 &= 690.147 \\ \ell_3 &= 690.1475 & \ell_8 &= 690.145 \\ \ell_4 &= 690.140 & \ell_9 &= 690.142 \\ \ell_5 &= 690.147 \end{aligned}$$

Let c be the constant portion of the instrument error. Then ℓ_3, which represents setting the instrument up once, will be the true length (neglecting the random portion of the error for the moment) t, plus the constant error c, ($\ell_3 = t + c$).

Similarly $\ell_2 = t + 2c$ and $\ell_1 = t + 4c$ because the instrument was set up two and four times, respectively, to complete the entire line. Subtracting we have

$$\begin{aligned} \ell_2 - \ell_3 &= c = 0.004 & c &= 0.004 \\ \ell_1 - \ell_2 &= 2c = 0.008 & c &= 0.004 \\ \ell_1 - \ell_3 &= 3c = 0.012 & c &= 0.004 \end{aligned}$$

Each time the instrument is set up, an error of 0.004 meters is added to the measured length. In column 3 of table 5, 0.004 meters has been subtracted from the lengths of column 2. The random portion of the error can now be estimated by calculating the length of the line from

$$\begin{aligned} \ell_4 &= AB + BC + CD + DE \\ \ell_5 &= ED + DC + CB + BA \\ \ell_6 &= AC + CE \\ \ell_7 &= EC + CA \\ \ell_8 &= AE \\ \ell_9 &= EA \end{aligned}$$

These lengths are also given in table 5 and have a standard deviation of 0.003 meters. The measurements of the five-station baseline have shown that the instrument error was composed of two parts; (1) a long-term offset error of 0.004 meters, and (2) a short-term error of 0.003 meter. The sum, $E = \sqrt{(0.004)^2 + (0.003)^2} = 0.005$ meter.

This is the magnitude of the error that might be expected from measurements made in the field on a given day.

Frequency. The (b) portion of the error in DME consists of the short- and long-term variations in the frequency standard used to control the light modulator. The only proper method of checking this frequency is by using a properly calibrated electronic counter. Some manufacturers provide a connector on the instrument so that the frequency may be easily monitored.

When purchasing an instrument, insist on a means of checking frequency. Many instruments do not use ovens for their crystals and may have rather large errors, particularly under extremes of temperature. The error is proportional to the length measured so that at short distances, it may not be detectable. If the instrument is being used to measure a precise baseline, an electronic counter should be used to measure the modulation frequency before and after the measurement.

ATMOSPHERIC CORRECTIONS. The accuracy of measurements made with DME depends not only upon the instrument itself but also upon a knowledge of the velocity of light along the measuring path. Techniques to be discussed later reduce this dependence to a minimum in the case of measuring movements in large structures, but in order to make absolute measurements, it is vital to have an accurate knowledge of the refractive index of the air along the line being measured. The velocity of light, the refractive index of the air, and the instrument itself are related in the following manner.

In a vacuum, light of all wavelengths travels at the same velocity, which has been carefully measured and found to be, 299,792.5 km per sec or 983,571,194 ft per sec (983,569,226 survey ft per sec). When not in a vacuum, light travels at a lower velocity determined by the temperature and pressure of the air and by the wavelength of the light. The velocity dependence on wavelength is

$$n_g = 1 + \left(287.604 + \frac{4.8864}{\lambda^2} + \frac{0.068}{\lambda^4} \right) . 10^{-6}$$

where n_g is the refractive index, and λ is the wavelength of the light expressed in micrometers for a temperature of 0° C and 760 mm of mercury (H_g). The subscript g refers to the fact that the light is modulated, which is the case for DME. The refractive index is simply a measure of how much light is slowed down in traveling through a medium other than a vacuum and is related to the vacuum velocity by $V = C/n_g$. The vacuum velocity is C, the velocity of light in air is V, and the refractive index is n_g. Some representative refractive indices used with modern DME are

Wavelength (micrometers)	Source	Refractive Index
0.48	Xenon arc	1.0003101
0.63	HeNe laser	1.0003002
0.84	GaAs Laser	1.0002947
0.91	GaAs Diode (infrared)	1.0002936

These refractive indices are at standard conditions. For any other conditions of temperature and pressure, a new refractive index may be found from

$$n_a = 1 + \left(\frac{n_g - 1}{1 + \frac{T}{273.2}}\right)\left(\frac{P}{760}\right)$$

where T is in °C, and P is in mm of H_g. Using these facts, the instrument manufacturer selects a standard set of conditions, about which he designs an instrument. He might, for example, design the DME to give a correct result at 15°C and 760 mm H_g, with an infrared diode. In this case, n_g = 1.0002936 and n_a = 1.0002783. In addition, a measuring interval of 10 meters might be selected to provide conveniently the accuracy intended by the manufacturer. Because DME measures round trip distance, a 20-meter modulation wavelength would be necessary to achieve the 10-meter measuring interval. The modulation wavelength, in turn, is related to the velocity of light through the modulating frequency by $f = V/\lambda_m$, where f is the modulating frequency, V is the velocity of light under the chosen conditions, and λ_m is the modulation wavelength, in this case 20 meters. The velocity is the vacuum velocity of light divided by the refractive index, or 299,792.5/1.0002783 = 299,709.1 km per sec. Thus, the modulating frequency would be 299,709.1/.020 km = 14.985455 mHz, and this is the operating frequency of the crystal supplied with the DME.

For the surveyor, these calculations have already been made, and the instrument has been supplied with the proper crystal in place. For the further convenience of the surveyor, the atmospheric corrections are supplied in the form of a handy circular slide rule, tables, or a simple nomogram, which gives a parts-per-million correction depending on temperature and pressure. This correction may be either dialed into the instrument or applied directly to the measured distance. The procedure in most cases would be to measure both temperature and pressure at each end of the line. The temperatures and pressures must then be meaned, and parts-per-million correction must be determined from the appropriate source. Then, this correction must be dialed into the instrument before a measurement can be made.

In the case of work on dams, this procedure could prove to be awkward. In most cases, it would be simpler to set the parts-per-million dial to zero and keep a record of temperature and pressure to be applied later as a correction. For example, if a line were measured to be 1469.328 meters with a *zero* parts-per-million correction dialed into the instrument and if the mean temperature and pressure gave a correction of +22 parts-per-million, then the correction would be (1469.328 x 22)/1,000,000 = +.032 meters and the corrected distance would be 1469.360 meters.

If a programmable calculator is available, an even simpler method is to have the calculator determine the correct length. The equations for correcting any optical distance measuring instrument are given in appendix B.

Measurement of Temperature and Pressure. When absolute accuracy in measuring is required, such as when a baseline is laid out, temperature and pressure measurements play a vital part. To give an idea of the magnitude of errors arising from the incorrect application of temperature and pressure corrections, a change of 1°C will cause roughly a one-part-per-million change in the observed distance. A change in atmospheric pressure of 2.5 mm (0.1 inches) of H_g will also cause a one-part-per-million change in distance. Of the two, temperature is the more difficult to measure correctly.

Pressure measurements can be made accurately in the field with a good aneroid barometer, which may be read to 0.01 inches of H_g. However, the aneroid barometer must be checked frequently against a mercury barometer. This check should be made as often as possible, perferably once a day. In an emergency, the sea level pressure may be obtained from the weather bureau of local airport. This pressure must then be corrected to a known elevation at which the barometer is placed for calibration. The equation for the change in pressure with elevation is $P_h = P_o\ (1\text{-}0.0000225571\ h)^{5.2561}$ where P_h is the pressure at an elevation of h meters and P_o is the sea level pressure.

Pressure measurements should be made at both ends of the line, and the mean of the two values used in the refractive index equation. If it is not possible to place barometers at both ends of the line, place the barometer at the instrument end, and use the elevations of the two ends together with the pressure measured at the instrument to calculate the pressure at the other end. The equation given above may again be used, with P_h being the unknown pressure at one end of the line, h the difference in elevation between the two ends, and P_o the pressure at the instrument. The algebraic sign of h may be determined by remembering that the higher elevation will have the lower pressure.

Temperature is much more difficult to measure properly. There are two reasons for this. First, the measuring equipment must be well shielded from the sun's radiation. One way of doing this is to enclose the thermometer in a reflective insulating shield. This, however, permits the heat to build up within the shield, and thus a small fan or some other means must be used to move air over the temperature sensing device so that the true air temperature is read. The second, and more serious, cause for error is that temperatures measured at the end points of a line near the ground are a poor indication of the true temperature along the line. Studies have shown that during the day, temperatures near the ground are much warmer than those 30 meters above the ground. A 5-degree centigrade difference is not uncommon. At night the reverse is true, temperatures near the ground are cooler than those above. Unfortunately, many lines to be measured are more than 30 meters above the gound over most of their lengths. Consequently, because serious errors occur from taking temperature measurements near the ground, serious errors are introduced when calculating the length. One means of reducing this error is to mount the temperature sensor on a telescoping fiberglass pole and elevate it as high as possible. However, even this measure does not solve the problem completely. Thus, temperature measurements are one of the major sources of error in the accurate determination of distance.

In measuring dams or other large structures, refractive index errors are less important because displacement values are needed, and therefore relative, rather than absolute, distances may be used. Special techniques for use on dams will be discussed later.

There are times, however, when an accurate measurement of distance is needed. This is the case for a baseline or where it is desired to give scale to a figure. For a baseline, choose a site where the terrain will enable temperature measurements to be made at the average height of the line above the ground, at least every 500 meters along the entire length of the line plus the end points. When calculating the temperature along the line, give the end point values a weight of one and the intermediate values a weight of two. Thus, for a 1,500-meter line,

Position		Temperature x Weight
Instrument End		21.5 x 1 = 21.5
500 meters		24.0 x 2 = 48.0
1,000 meters		23.0 x 2 = 46.0
Reflector End		22.5 x 1 = 22.5
	Sum	138.0
138.0/6 =	Weighted mean temp	= 23.0C

form the measurements three times, about 4 hours before sunset, 1 hour before sunset, and 2 hours after sunset. If agreement between the three sets is satisfactory, the baseline has been accurately measured (assuming the instrument is working properly).

It may be difficult to measure temperature in the desired manner because the end points of the line are elevated and the intervening terrain is much lower. In this case, elevate the end point temperature measuring devices as high as possible, take a measurement 1 hour after sunrise, and take a second measurement 1 hour before sunset. The best condition for measuring baselines is an overcast day, with moderate winds to mix the air near the ground. As mentioned before, pressure measurements need to be taken only at the end points.

A correction for humidity has not been mentioned because the errors owing to water vapor are almost always small in comparison with temperature errors at optical wavelengths.

MEASUREMENTS OF LARGE STRUCTURES

RATIOS. It was shown previously that refractive index errors (temperature- and pressure-measuring errors) limit the accuracy of DME. Also, when temperature and pressure are measured properly, errors still occur because it is difficult or impossible to measure other than at the end points of the line. Further, refractive index measurements are both time consuming and expensive. This section discusses techniques for reducing refractive index errors in measurements of large structures by using ratios, or reference lines.

A number of studies, made as a part of a research program in the most accurate use of DME, have resulted in the formulation of two important experimental rules:

1. Refractive index errors, resulting from end point measurements of temperature and pressure, tend to be the same for all lines measured from one point within a short period of time.

2. The ratios of observed distances, measured from one point within a short period of time, are constant.

For both rules, a short period of time is 30 minutes or less.

In figure 4, lines AB and AC are measured from a common point. Rule 1 states that if refractive index measurements are made at points A, B, and C within a short period, the errors in the measurements tend to be the same at all three points. If the true temperature along line AB is 20°C, but the mean of measurements made at A and B is 24°C (a condition typical of daytime), then the mean of temperature measurements at the end points of line AC would also be expected to be 4°C higher than the true temperature along that line. Because 1°C is approximately equivalent to one-part-per-million of distance, both lengths will be in error by 4 ppm. However, if the measured length of AB is divided by the measured length of AC, the resulting ratio will equal the ratio of the true lengths. Thus, the *ratio* of two measured lengths will be more accurate than either of the lengths that were used to form the ratio. For example, AB was measured to be 2839.611 meters, and AC was measured to be 2241.487 meters. Their ratio is AB/AC = 1.26684250. Both lines were in error by 4 ppm because of temperature-measuring errors; therefore, the true lengths were, AB = 2839.611 + 0.0114 (4 ppm) and AC = 2241.487 + 0.0090 (4 ppm). The ratio of the true lengths is 2839.6224/2241.4960 = 1.26684250, the same as the ratio of measured lengths.

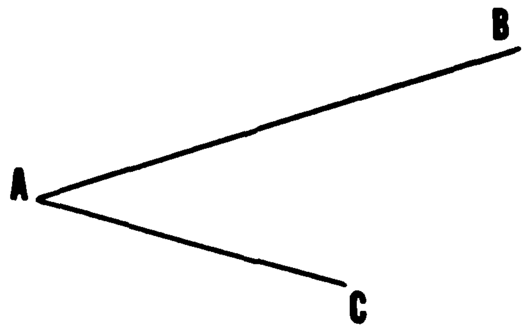


Figure 4. Ratio of Two Lines.

When ratios are formed from measurements that have had refractive index corrections applied, they will be called *corrected ratios.* The property of the corrected ratio is that it is very accurate. From corrected ratios, angles may be computed that are frequently within a few tenths of an arc second of their true values.

A second set of ratios can be obtained from the same measurements by using the data before the application of the refractive index corrections. These are called *observed ratios*, and they have been formed from lines that have had no temperature or pressure corrections applied. Rule two states that the observed ratio is constant. This means that the observed ratio of two lines measured today will agree with the observed ratio of the same two lines measured months or years later. This will be true even though the observed lengths of the individual lines have changed greatly because of changes in atmospheric conditions between the two sets of measurements.

The observed ratios will not, however, be the same as the corrected ratios unless certain conditions are met. To understand this, let us assume for a moment that an instrument has been set upon a hilltop. In the valley below, two points have been selected that are equidistant from the hilltop stations and are at the same elevation. The observed distances to the two points would appear the same because the distances are equal and both lines pass through roughly the same atmosphere. A point is then selected that is the same distance from the hilltop station as the other points, but with a higher elevation. When the observed distances are recorded, the two lengths to the valley points are the same, but the observed length to the higher elevation point is shorter. Because air density decreases with elevation, the light traversing the higher line travels faster and returns sooner. The instrument then shows the distance to be shorter. Two lessons can be learned from this. The first lesson is that if the mean elevations of two lines measured from a point are the same, the ratio of the observed distances is equal to the ratio of the corrected distances. In the example above, the observed distances to the valley points are the same, and the ratio of the two observed lengths is 1. The true lengths to the two points are the same so that the ratio of the corrected lengths is also 1. This is often the case with dams where the alignment markers along the crest of the dam are all within a few meters of the same elevation. This property of *observed ratios* will be used later on.

The second lesson is that when the elevations of the end points to which measurements are being made are different, the ratio of observed lengths is not the same as the ratio of corrected (true) lengths because the refractive indices along the two lines are different. Even though it is not accurate, the observed ratio does not change with time and it may be used to detect changes in position. Furthermore, the observed ratio may be corrected by means of an atmospheric model (appendix C).

In many respects, ratios have properties similar to those of angles. In triangulation, the sum of the three angles of a triangle must equal 180°, and a knowledge of two angles permits calculation of the third. Similarly, the product of three ratios obtained from a triangle must equal 1, and a knowledge of two ratios permits calculation of the third. In figure 5, the triangle shown has sides A, B, and C as measured from vertices 1, 2, and 3. The ratio measured from vertex 1 is A_1/B_1, using a counterclockwise convention (A_1/B_1 rather than B_1/A_1) with the subscript designating the vertex from which the ratio was measured. Two other ratios, B_2/C_2 and C_3/A_3, may also be measured. If the measurements are perfect, $A_1 = A_3$, $B_1 = B_2$, $C_2 = C_3$ and $A_1/B_1 \times B_2/C_2 \times C_3/A_3 = 1$. If the measurements are not perfect (the usual case), the degree to which the product failed to equal 1 is a measure of the precision of the measurements. If only two ratios were measured, the third may be calculated. For example, $A_1/B_1 = C_2/B_2 \times A_3/C_3$. Angles may be calculated directly from the ratios by using a modified cos formula:

$$\cos \angle 1 = \tfrac{1}{2}\left[\frac{A_1}{B_1} + \frac{B_1}{A_1} - \left(\frac{C_3}{A_3} \times \frac{C_2}{B_2}\right)\right]$$

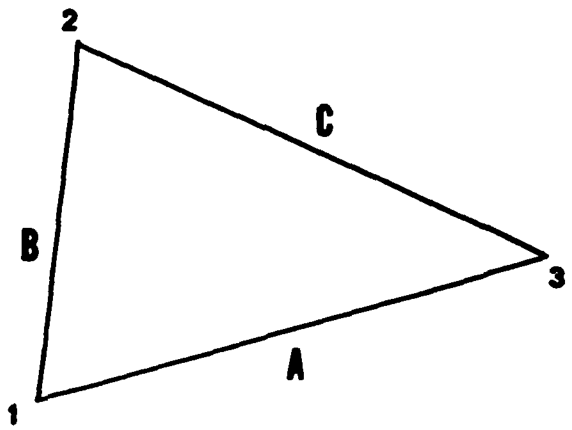


Figure 5. Ratios in a Triangle.

Formulas for the remaining angles are given in appendix A. The use of ratios yields angles as a result, and the angles determined from the ratios are more accurate than those determined from the lengths alone because a ratio is more accurate than either of the lengths from which it is derived.

When the angles of a triangle do not sum to 180°, the triangle may be adjusted by taking one-third of the difference between 180° and the sum of the angles and by applying it as a correction to each angle. With ratios, a correction may be made to each ratio. An adjustment of the angles of a triangle is given in appendix D.

Movement Measurements in Large Structures. Measurements of movements in large structures can be made very accurately, in two dimensions, by using trilateration techniques. The work consists of two phases, the control network and the structure itself.

The Control Network. In monitoring possible movements of structures, points on the structure must be related to points that have been selected for stability, usually at some distance from the structure itself. These will be called control points, and all movements of the structure will be related to one or more of them. It is important that the control points not move, and for this reason, they should be placed in geologically stable positions. They should also afford a good geometry for trilateration measurements. Good geometry, in turn, consists of measuring along the line where movement is expected. For example, if measurements of upstream or downstream movements are required, the control point should be located either upstream or downstream. Further, the point should be at a sufficient distance from the structure so that the end points, as well as the center, can be monitored with good geometry. In figure 6, a dam is shown with both an upstream and downstream control monument. Geometrically, measurements from the upstream side of the dam will be poor, while those from the downstream side will be much stronger. If movement in two dimensions is desired, a point off the end of the dam should also be chosen (see figure 6). For best results, the angle of intersection (θ) should be 90°. A technique for selecting the best control points and for estimating the magnitude of measuring errors to points on the structure is given in appendix E. Two selections of control figures are shown in figure 7.

A final criteria for the selection of control monuments is intervisibility. Because the control figure also provides a means of correcting for refractive index, the points selected for control at the ends of the dam must be visible from the upstream and/or downstream points.

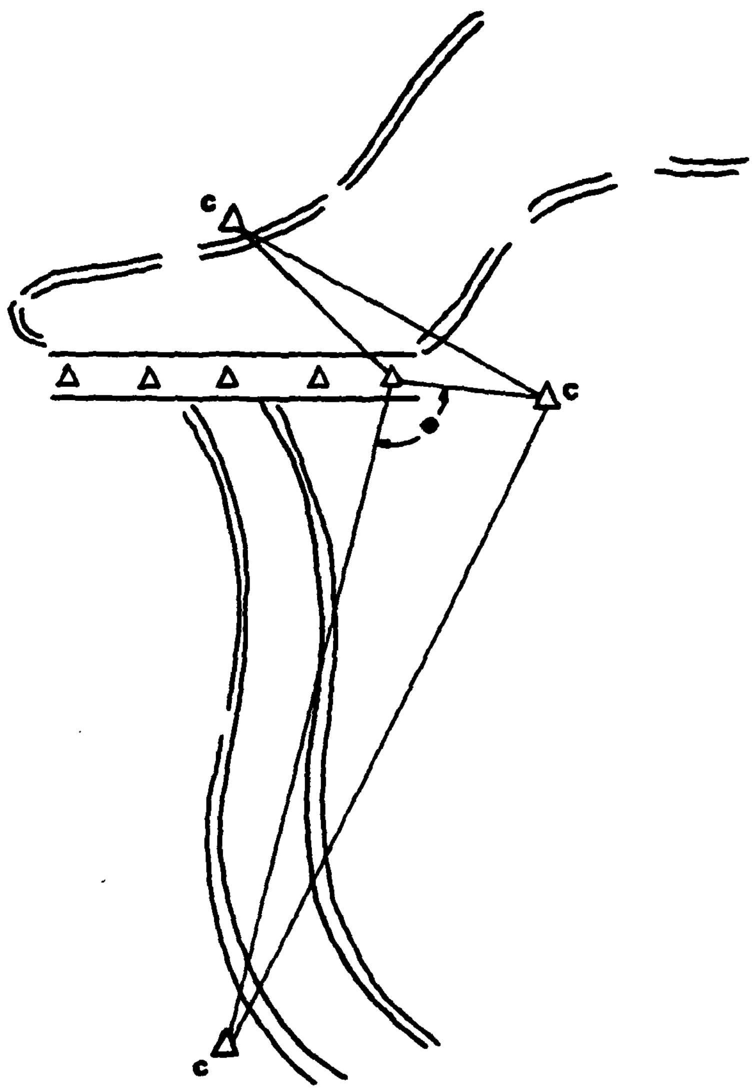


Figure 6. Control Monuments for a Dam.

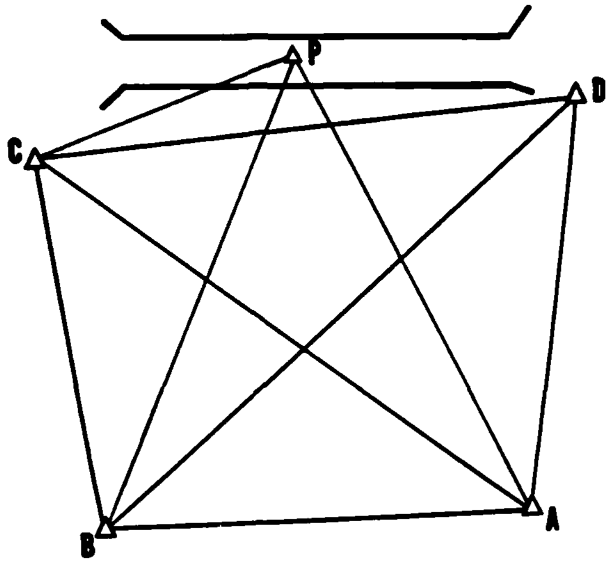


Figure 7a. A Strong Control Figure.

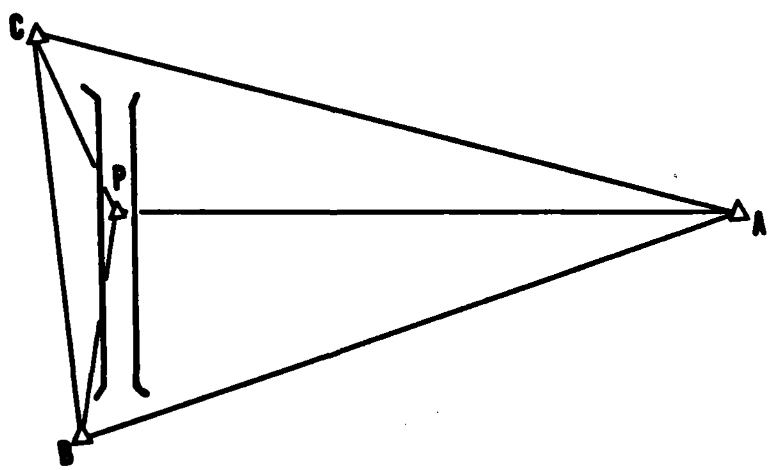


Figure 7b. A Simple Control Figure.

In trilateration, lengths to an unknown station from each of two control points will give the position of the unknown station in two dimensions. Measurements from three control stations will give three positions of the unknown station, and may be used as a check of survey accuracy. Figure 7a shows a good control figure for the measurement of a dam. In the control figure, A, B, C, and D are control monuments. All are intervisible. Point P is an unknown station on the dam and is measured from control points A, B, and C. Positions of P are calculated from measurements of lines AP and BP, from lines BP and CP, and from lines AP and CP. The agreement between the three positions obtained for point P is a measure of the accuracy of the survey.

When measurements are made of lines exceeding 600 meters, a major source of error is the inability to determine accurately the refractive index along the line. An error in temperature of 1° C or in pressure of 2.5 mm (0.1 inches) of H_g will cause an error in length of one part per million. These errors may be minimized by considering the ratio of two lines that have been measured within 30 minutes of each other. The errors of each line tend to be the same so that taking a ratio greatly reduces the magnitude of the error. This may be shown by again referring to figure 7a. Point D has been selected as a reference point. Its position was chosen so that it would be in stable ground, it would be visible from the other control points, and the lines to it from the other control points would pass through similar atmospheric conditions to those from the control points to unknown positions on the dam.

The first time a dam is visited to make trilateration measurements, both ratios and conventional measurements are made to determine the shape and size of the control figure. The simplest example would be the triangular figure shown in figure 7b. All control monuments should be occupied by the DME. At each point, measurements should be made to all of the other control monuments within a short period of time. In the case of the triangle ABC in figure 7b, monument A would be occupied and lengths AC and AB measured. Careful refractive index readings should also be taken at both ends of each line as it is measured. Similar measurements should then be made as the DME occupies stations B and C. Each line should then be reduced to the level or spheroid and have the refractive index corrections applied. A typical set of measurements for triangle ABC is shown on the following page. Experience has shown that the adjusted angles (see appendix D) determined from corrected ratios are more accurate than the angles determined from the means of the lengths of the sides because ratios are more accurate than the lengths of which they are composed.

It may be seen from this example that the result of working with ratios is angles, and that in effect very accurate triangulation is being carried out using DME. As in the case of triangulation, a base line is necessary to determine the scale when ratios are used.

Length		Ratio
A to C	2547.447	AC/AB 0.9181349
B	2774.589	
B to A	2774.583	BA/BC 3.7776155
C	734.480	
C to B	734.478	CB/CA 0.2883212
A	2547.430	

$$\frac{AC}{AB} \times \frac{BA}{BC} \times \frac{CB}{CA} = 1.0000018$$

Adjusted Angles

A	15°	05′	47″.84
B	64°	35′	55″.08
C	100°	18′	17″.08

By way of comparison, angles calculated from the mean lengths would be:

A	15°	05′	47″.59
B	64°	35′	53″.76
C	100°	18′	18″.65

Note: lengths are spheroid distances in meters.

Choose one of the sides of the triangle to serve as a baseline, and use the mean length as the scale for the triangle. In the example, AB has been chosen and its length is 2774.586 meters. Next, by using the sine formula and the angles determined from ratios, the other two sides may be determined.

$$\frac{2774.586}{\sin C} = \frac{BC}{\sin A} = \frac{AC}{\sin B}$$

$BC = 734.481$ $\qquad$ $AC = 2547.443$

Although these calculations may seem laborious, the angles obtained are of the highest accuracy. The scale, however, is only as accurate as the mean of the two measurements of the baseline. Fortunately this is not a serious problem with measurements of dams because changes in lengths are desired rather than the absolute lengths themselves.

The final task in establishing the control network is to assign coordinates to A, B and C. These may be fitted into an existing network, or a local control net may be set up for the project.

At some later date, the control figure may once again be occupied. The same procedure may be used, and the angles determined and compared with those obtained during the first survey. This, however, requires the use of temperature- and pressure-measuring devices each time the figure is surveyed.

An easier method is to use the observed ratios, for these do not require knowledge of the refractive index. Remember that the observed ratios remain constant, and thus comparison of observed ratios from the first survey with observed ratios from the second survey are sufficient to determine whether any of the control monuments have moved. In fact, measurements of temperature and pressure need only be made of the control lines in order to give the proper scale to the figure. And these measurements need only be made the first time a project is surveyed. From that time on, only observed distances are required. In addition, all of the measurements from the control monuments to stations on the dam will be observed distances. Measurements of temperature and pressure are not necessary.

Points On The Dam. When positions have been established for the monuments in the control figure, observed ratios will be used to determine the refractive index corrections for measurements of points on the dam. Referring again to figure 7b, the lines AC, AB and BC have been corrected for refractive index and may be used as reference lines. For measurements from control monument A, either AC or AB may be used as a reference line. A good reference line is one which traverses approximately the same atmosphere as is found along the lines to points on the dam and is almost the same length or longer. If we call the corrected length of the reference line R_{corr} and the observed length of the same line R_{obs}, the following equation may be written

$$R_{obs} \times k = R_{corr}$$

Where k is a constant owing to the atmospheric conditions along the line at the time it was measured. Because the reference line has been selected to travel through approximately the same atmosphere as that to points on the dam, we may say that k is also the atmospheric constant for lines measured to the dam. If P_{obs} is the observed length to a point on the dam, then the corrected distant, P_{corr}, may be found from

$$P_{obs} \times k = P_{corr}$$

This technique enables the surveyor to correct for refractive index without using temperature- and pressure-measuring equipment. However, k is not really a constant. It changes slowly with time. For this reason, it must be remeasured at approximately 30-minute intervals, and it must be assumed that it changes in a linear fashion. The following example will show how this might be done.

In figure 8, the DME has been set up at A. Measurements are made of AC, AP_1, AP_2, AP_3, and again AC. After the observed lengths have been reduced to the level or the spheroid, the following measurements from control monument A are listed:

To Sta	Time	Observed Length (D_{obs})	Refractive Index Constant (k)	Corrected Distance (D_{corr})
C	1330	2547.326	1.0000459	2547.443*
P_1	1335	2477.075	1.0000454	2477.187
P_2	1340	2407.354	1.0000449	2407.462
P_3	1345	2445.152	1.0000445	2445.261
C	1350	2547.331	1.0000440	2547.443*

* Note: AC is the reference line

The first and last measurements are of AC. The length of AC is known and is used as a reference line to calculate the value of the refractive index constant. At first, the constant was 1.0000459 (2547.443/2547.326), but because of changes in the atmosphere, it changed to 1.0000440 (2547.443/2547.331). The value of k at intermediate times may be found by assuming that the change was linear. Thus, a value of k may be found for the times when P_1, P_2, and P_3 were measured. Applying the appropriate value of k to the observed length (D_{obs}) of AP_1 gives 2477.075 x 1.0000454 = 2477.187 as its corrected length (D_{corr}).

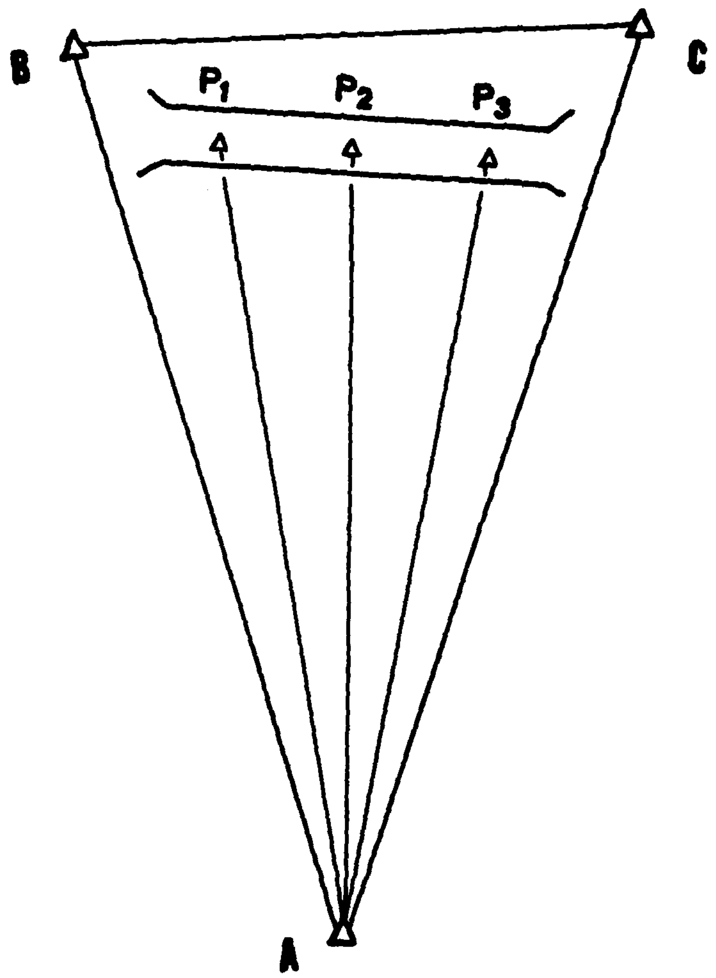


Figure 8. Use of a Reference Line.

Any length in a control figure may serve as a reference line, although some lines will be better than others. From A, AB would also serve. From B however, BC would be a better choice than BA because it passes through a similar atmosphere to that found in measuring from B to P_1, P_2, and P_3.

Reduction To The Spheroid. Mention has been made of reducing lines either to the level or the spheroid. In very accurate work where lines exceed 1 km, the surface upon which a survey is being made can no longer be considered a plane. If distances are reduced to the level and used to calculate angles, the angles thus obtained may not agree with angles obtained from a theodolite. Further, the position of a point calculated from the lengths to two control monuments may not agree with the position of the same point when measured from two other control monuments. To prevent problems of this type, figures with line lengths in excess of 1 km should be reduced to the spheroid instead of the level. The equation to be used is

$$D_s = R \sqrt{\frac{[D_1 - (e_2 - e_1)][D_1 + (e_2 - e_1)]}{(R + e_2)(R + e_1)}}$$

where

D_s = Spheroid chord distance
R = Earth radius
D_1 = Observed distance from the DME
e_1 = Elevation + H.I. of the instrument
e_2 = Elevation + H.I. of the reflector

for the earth radius, use 6,372,000 meters. In the next section, all lines will be reduced to the spheroid.

Examination of A Fictitious Dam. Previous sections of this report have dealt with using DME properly, with making corrections for refractive index, and with using ratios. This section will combine these elements to show how they may be used for a precise survey of a dam.

A concrete dam is portrayed in figure 9. Control pedestals have been set at points C1, C2, C3, and C4. Markers A1 through A6 have been set along the crest of the dam, and T1 and T2 have been set near the toe of the dam. Elevations have been measured to obtain the list in table 6.

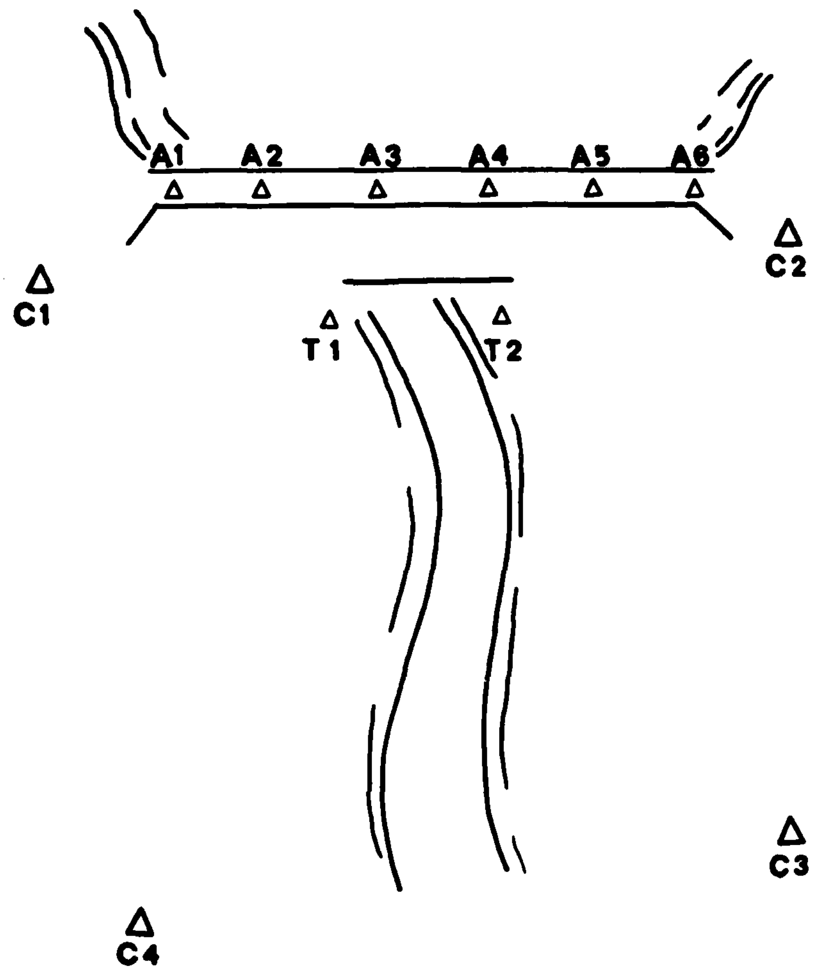


Figure 9. A Fictitious Dam.

Table 6.	Elevations
A1	410.724
A2	410.718
A3	410.706
A4	410.721
A5	410.712
A6	411.245
C1	419.911
C2	413.275
C3	463.701
C4	521.537
T1	329.623
T2	329.394

Note: Elevation measurements in meters above sea level.

Each of the control monuments were occupied with the DME, and measurements were made to the other three control monuments. Temperatures and pressures were also taken at both ends of these lines. After measuring the control lines, the lengths to stations on the dam were measured from three of the control monuments. Temperatures and pressures were not taken for these lines.

On 2 February, the following lengths were measured from C3 (see table 7). Measurements began with the control figure. Either C1 or C2 could have been used for a reference line, but in this case C1 has been chosen. Because it was the reference line, it was measured before and after the remaining control lines. This practice helped to check for both drift in the instrument and in the atmospheric conditions. When the control lines were completed, the operator next measured to points on the dam. Forty minutes had elapsed after completion of the control line measurements before the field party with reflectors were set up on the dam. Because the reference line should be measured approximately every 30 minutes, the observed distance to C1 was again measured (measurement 5). A reflector was left unattended at C1 because it was no longer necessary to read the temperature and pressure. Remember temperature and pressure measurements are made only on the control lines and only when a study is made for the first time at a particular dam. The next time the dam is visited, perhaps 6 or 12 months later, it will not be necessary to measure refractive index. Possible movement in the control figure may be checked at that time by a comparison of ratios of observed distances.

Table 7. Measurements from C3.

Meas	To	Time	Observed Dist. D_{slope} (meters)	Mean Temp (°C)	Mean Press (in. H_g)
1	C1	0930	1081.105	16.4	28.26
2	C4	0935	945.032	16.4	28.09
3	C2	0940	703.788	17.0	28.27
4	C1	0945	1081.104	16.7	28.26
5	C1	1025	1081.101		
6	A1	1035	968.241		
7	A2	1045	924.456		
8	C1	1050	1081.103		
9	A3	1100	882.721		
10	A4	1115	843.323		
11	C1	1120	1081.104		
12	A5	1130	806.626		
13	A6	1145	772.950		
14	C1	1150	1081.104		
15	C1	1300	1081.100		
16	T1	1305	872.886		
17	T2	1315	836.021		
18	C1	1320	1081.097		

Measurements were made that same afternoon from C1. Only the control lines were measured. Three sets of positions will be obtained for the stations on the dam from C2, C3, and C4. Measurements from C1 would do little to improve the accuracy of these positions in the upstream-downstream direction. Table 8 gives the lengths from C1 (taken on 2 February).

Table 8. Measurements from C1.

Meas	To	Time	Observed Dist D_{slope} (meters)	Mean Temp (°C)	Mean Press (in. H_g)
19	C2	1400	566.212	19.0	28.28
20	C3	1405	1081.095	18.8	28.20
21	C4	1410	989.418	18.5	28.09
22	C2	1415	566.215	18.8	28.28

Note: A week later, monument C4 was occupied (measurements taken 9 February).

Table 9. Measurements from C4.

Meas	To	Time	Observed Dist D_{slope} (meters)	Mean Temp (°C)	Mean Press (in. H_g)
23	C1	0835	989.446	6.1	28.85
24	C2	0840	1138.277	6.1	28.87
25	C3	0845	945.050	5.8	28.78
26	C1	0850	989.445	6.2	28.85
27	C1	0900	989.444		
28	A1	0905	1031.587		
29	A2	0915	1042.973		
30	A3	0925	1057.756		
31	C1	0930	989.438		
32	A4	0940	1075.788		
33	A5	0945	1096.925		
34	A6	0955	1120.924		
35	C1	1000	989.432		
36	T1	1010	981.303		
37	T2	1020	987.682		
38	C1	1025	989.431		

Note: Finally, measurements were made from monument C2 on 9 February.

Table 10. Measurements from C2.

Meas	To	Time	Observed Dist D_{slope} (meters)	Mean Temp (°C)	Mean Press (in. H_g)
39	C1	1230	566.225	8.1	29.04
40	C4	1235	1138.273	7.6	28.87
41	C3	1240	703.799	7.8	28.97
42	C1	1245	566.225	8.3	29.04
43	A1	1250	398.146		
44	A2	1300	337.350		
45	A3	1310	276.652		
46	C1	1315	566.225		
47	A4	1320	216.070		
48	A5	1330	155.828		
49	A6	1335	96.436		
50	C1	1345	566.224		

Note: From C2, measurements could not be made to T1 and T2, the stations at the toe of the dam. This completed the field measurements.

The first step in the data reduction is to bring all the lines (D_{slopes}) down to the spheroid. The equation in appendix A plus an earth radius of 6, 372, 000 meters have been used to obtain the values in table 11. Elevations are given in table 6. For the sake of simplicity, the lengths in tables 7 through 10 have already been reduced to mark-to-mark distances so that the heights of the instrument and reflector above the mark are zero. Column 4 of table 11 lists the observed spheroid distances (D_{obs}) for the measurements of tables 7 through 10. Column 5 lists the corrected spheroid distances (D_{corr}). Those lengths in column 5 that are followed by an asterisk have been corrected for refractive index by temperature and pressure measurements made at the time the line was measured. The instrument used for the measurements contains a He Ne laser with a wavelength of 0.6328 micrometers and gives a correct reading of distance at 20°C and 760 mm of H_g. Appendix B gives the equations necessary for refractive index corrections.

Table 11. Corrected Line Lengths

(1)	(2)	(3)	(4)	(5)
Meas	C3 To	Time	D_{obs} (meters)	D_{corr} (meters)
1	C1	0930	1080.143	1080.156*
2	C4	0935	943.188	943.201*
3	C2	0940	701.931	701.940*
4	C1	0945	1080.142	1080.155*
5	C1	1025	1080.141	(1080.155)
6	A1	1035	966.724	966.736
7	A2	1045	922.873	922.884
8	C1	1050	1080.141	(1080.154)
9	A3	1100	881.068	881.078
10	A4	1115	841.599	841.609
11	C1	1120	1080.142	(1080.154)
12	A5	1130	804.828	804.837
13	A6	1145	771.115	771.124
14	C1	1150	1080.142	(1080.154)
15	C1	1300	1080.138	(1080.154)
16	T1	1305	862.473	862.486
17	T2	1315	825.111	825.125
18	C1	1320	1080.135	(1080.154)

Table 11. Continued

(1) Meas	(2) C1 To	(3) Time	(4) D_{obs} (meters)	(5) D_{corr} (meters)
19	C2	1400	566.136	566.144*
20	C3	1405	1080.133	1080.149*
21	C4	1410	984.112	984.128*
22	C2	1415	566.139	566.147*
	C4 To			
23	C1	0835	984.140	984.137*
24	C2	0840	1133.034	1133.030*
25	C3	0845	943.206	943.203*
26	C1	0850	984.139	984.136*
27	C1	0900	984.138	(984.134)
28	A1	0905	1025.543	1025.540
29	A2	0915	1036.993	1036.992
30	A3	0925	1051.857	1051.858
31	C1	0930	984.132	(984.134)
32	A4	0940	1069.987	1069.991
33	A5	0945	1091.232	1091.238
34	A6	0955	1115.403	1114.411
35	C1	1000	984.126	(984.134)
36	T1	1010	962.289	962.297
37	T2	1020	968.747	968.756
38	C1	1025	984.125	(984.134)
	C2 To			
39	C1	1230	566.149	566.147*
40	C4	1235	1133.030	1133.027*
41	C3	1240	701.942	701.940*
42	C1	1245	566.149	566.147* (566.146)
43	A1	1250	398.112	398.110
44	A2	1300	337.318	337.316
45	A3	1310	276.622	276.621
46	C1	1315	566.149	(566.146)

Table 11. Continued

(1)	(2)	(3)	(4)	(5)
Meas	C2 To	Time	D_{obs} (meters)	D_{corr} (meters)
47	A4	1320	216.041	216.040
48	A5	1330	155.797	155.796
49	A6	1335	96.408	96.408
50	C1	1345	566.148	(566.146)

Note: * Length corrected from temperature and pressure measurements.
( ) True Length

Those lengths in column 5 that do not have an asterisk following them or which are in parenthesis will be explained later.

When the lines have been reduced to the spheroid, the next step is to define the size and shape of the control figure, in this case a doubly braced quadralateral. There are several ways to do this. One way is that the figure contains four triangles, and these may be individually treated in the same manner as the triangle in figure 7b. Another way would be to use the means of the six lines in the figure and adjust these by means of the quadralateral adjustment given in appendix D. This is the technique that was used in the present case to obtain the adjusted lengths given below.

C1	to	C2	566.146 meters
C1	to	C3	1080.154
C1	to	C4	984.134
C2	to	C3	701.940
C2	to	C4	1133.029
C3	to	C4	943.202

The control figure may be fit into an existing coordinate system or a local system may be devised just for the dam. For the fictitious dam, a local system was used. C4 was selected as a starting point and was assigned coordinates of

$$C4 \quad x = 1000.000$$

$$y = 1000.000$$

The coordinates of C3 were then chosen to place C3 at a distance of 943.202 meters from C4.

$$\text{C3} \quad x = 1943.202 \qquad y = 1000.000$$

The placement of C4 and C3 has determined the scale and orientation of the figure. Using the positions of C3 and C4 and the appropriate lengths, the positions of C1 and C2 may be determined by the equations in appendix A.

$$\text{C1} \quad x = 1366.527 \qquad y = 1913.333$$

$$\text{C2} \quad x = 1890.936 \qquad y = 1699.991$$

The extablishment of the control figure needs be done only once. From that time on, it is only necessary to check for movements of the control monuments. This may be done by comparing observed ratios taken at some later time with the original set.

Returning to table 11, one may now calculate the corrected lengths (D_{corr}) to the stations on the top and toe of the dam from the control monuments. This is done by using reference lines to make refractive index corrections.

Measurements 15 through 18 from table 11 are given in table 12.

Table 12. Changes of Correction Factor with Time

Meas	C3 To	Time	D_{obs} (meters)	Correction Factor	D_{corr} (meters)
15	C1	1300	1080.138	1.0000148	(1080.154)
16	T1	1305	862.473	1.0000155	862.486
17	T2	1315	825.111	1.0000169	825.125
18	C2	1320	1080.135	1.0000176	(1080.154)

At 1300, when the distance to C1 was measured, the observed distance (D_{obs}) was found to be 1080.138 meters. This line, C3 to C1, is a part of the control figure, and its correct length has been determined to be 1080.154 meters. The atmospheric correction at 1300 may then be found by dividing. The correction is 1080.154/1080.138 = 1.0000148. Later, at 1320, the atmospheric correction has become 1.0000176. Assuming the change in correction has been linear as a function of time over the 20-minute interval, we may calculate the correction factor at 1305 and 1315 when observed distances were measured to T1 and T2. Multiplying the observed distance by the corresponding atmospheric correction gives the corrected distance (D_{corr}) to T1 and T2. Thus, in table 11, the values in parenthesis in column 5 are the correct, or true, lengths of reference lines, and the values without an asterisk or parenthesis are the corrected lengths that have been calculated from reference lines.

Finally, with the corrected lengths and the coordinates of the control monuments from which they were measured, it is possible to calculate the positions of the points on the dam. The appropriate equations are given in appendix A. Because three lengths were measured to stations on the crest of the dam, three solutions will be obtained. Geometrically, some solutions will be superior to others. To obtain the best solution, the adjustment technique given in appendix D was used. For stations at the toe of the dam, only one solution is possible.

In table 13, positions of the crest and toe markers are given for various line combinations. In the case of the crest markers, an adjusted position is also given.

Table 13. Crest and Toe Station Positions

Station	x	y	From
A1	1533.713	1875.726	C2 – C3
	1533.710	1875.720	C2 – C4
	1533.705	1875.723	C3 – C4
	1533.709	1875.722	Adjusted
A2	1590.161	1852.688	C2 – C3
	1590.158	1852.682	C2 – C4
	1590.153	1852.685	C3 – C4
	1590.157	1852.684	Adjusted

Table 13. Continued

Station	x	y	From
A3	1646.583	1829.648	C2 – C3
	1646.588	1829.656	C2 – C4
	1646.594	1829.652	C3 – C4
	1646.589	1829.653	Adjusted
A4	1703.041	1806.615	C2 – C3
	1703.038	1806.609	C2 – C4
	1703.033	1806.613	C3 – C4
	1703.037	1806.612	Adjusted
A5	1759.465	1783.585	C2 – C3
	1759.467	1783.588	C2 – C4
	1759.470	1783.584	C3 – C4
	1759.468	1783.586	Adjusted
A6	1815.919	1760.547	C2 – C3
	1815.915	1760.542	C2 – C4
	1815.912	1760.545	C3 – C4
	1815.915	1760.544	Adjusted
T1	1568.152	1776.672	C3 – C4
T2	1608.187	1754.053	C3 – C4

If desired, alignment may be determined from positions. Using the crest stations A1 and A6 as end points, the alignment of A2 through A5 is given in table 14. T1 and T2 are also included in the alignment to help monitor any tilt in the dam. Alignment done from positions is not affected by curved dams, by bends, or by differences in elevations. The equations used for alignment are given in appendix A.

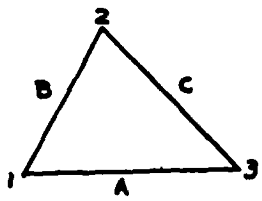
Table 14. Alignment

Station	Dist from A1 (meters)	Dist off Line (meters)
A2	60.968	0.000
A3	121.919	– 0.001
A4	182.888	+ 0.001
A5	243.836	– 0.004
T1		+ 78.691
T2		+ 84.505

Note: + = Downstream
– = Upstream

APPENDIX A. GENERAL EQUATIONS FOR USE WITH TRILATERATION

Calculations of Angles From Ratios



$$\cos\angle 1 = \frac{1}{2}\left[\frac{A_1}{B_1} + \frac{B_1}{A_1} - \left(\frac{C_3}{A_3} \times \frac{C_2}{B_2}\right)\right]$$

$$\cos\angle 2 = \frac{1}{2}\left[\frac{B_2}{C_2} + \frac{C_2}{B_2} - \left(\frac{A_1}{B_1} \times \frac{A_3}{C_3}\right)\right]$$

$$\cos\angle 3 = \frac{1}{2}\left[\frac{C_3}{A_3} + \frac{A_3}{C_3} - \left(\frac{B_2}{C_2} \times \frac{B_1}{A_1}\right)\right]$$

The subscripts 1, 2, and 3 refer to the vertices from which the lengths were measured.

EXAMPLE

$A_1 = 1198.106$ $\quad B_2 = 1631.099$ $\quad C_3 = 1541.164$

$B_1 = 1631.110$ $\quad C_2 = 1541.141$ $\quad A_3 = 1198.112$

$$\begin{aligned} \angle 1 &= 63^\circ\ 52'\ 42''.02 \\ \angle 2 &= 44^\circ\ 16'\ \ 1''.59 \\ \angle 3 &= 71^\circ\ 51'\ 16''.39 \\ \Sigma &= 180^\circ\ 00'\ 00''.00 \end{aligned}$$

Reduction Of A Slope Distance to A Chord Distance On The Spheriod

$$D_s = R\sqrt{\frac{[D_1 - (e_2 - e_1)]\ [D_1 + (e_2 - e_1)]}{(R + e_2)\ (R + e_1)}}$$

Where

D_s = Spheroid chord distance
R = Earth radius
D_1 = Observed distance from the DME
e_1 = Elevation + H.I. of the instrument
e_2 = Elevation + H.I. of the reflector

R = 6,372,000 meters for lines less than 5 kilometers in length. For lines in excess of 5 kilometers, a more precise radius must be used as determined from the following equation:

$$R = \frac{6335035}{1 - 0.010137 \sin^2 \phi - 0.006769 \sin^2 \alpha (1-\sin^2 \phi)}$$

where

R = Earth radius in meters
ϕ = Latitude of line
α = Azimuth of line

EXAMPLE

R = 6365237 meters
ϕ = 39°
α = 25°

D_1 = 2085.304 meters
e_1 = 341.202
e_2 = 286.118

D_s = 2084.474 meters

Intersection Of Two Known Lengths From Known Positions.

$$X_3 = X_1 + \ell_1 \sin(\theta - \phi)$$
$$Y_3 = Y_1 + \ell_1 \cos(\theta - \phi)$$

X_1, Y_1 = Coordinates of first known position
X_2, Y_2 = Coordinates of second known position
X_3, Y_3 = Coordinates of unknown position
ℓ_1 = Distance from X_1, Y_1 to unknown position
ℓ_2 = Distance from X_2, Y_2 to unknown position
ℓ_0 = Distance between known positions

$$\ell_0 = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$$

$$\phi = \text{Cos}^{-1} \frac{\ell_0^2 + \ell_1^2 - \ell_2^2}{2\,\ell_0\,\ell_1}$$

$$\theta = \tan^{-1} \frac{X_2 - X_1}{Y_2 - Y_1} \quad *$$

* The sign and quadrant of θ will depend on the signs of the numerator and denominator of the quantity in parenthesis.

$$\tan^{-1} \frac{+1}{+1} = +45^\circ \qquad \tan^{-1} \frac{-1}{-1} = -135^\circ$$

$$\tan^{-1} \frac{-1}{+1} = -45^\circ \qquad \tan^{-1} \frac{+1}{-1} = +135^\circ$$

This result may also be obtained by using the rectangular to polar coordinate key on a calculator.

This problem has two solutions. The second solution may be obtained by interchanging X_1 and X_2, Y_1 and Y_2 and ℓ_1 and ℓ_2, and reworking the problem.

EXAMPLE

X_1	= 26.073	X_2	= 147.747
Y_1	= 95.601	Y_2	= 17.382
ℓ_1	= 179.169	ℓ_2	= 132.377

	1st Solution	2nd Solution
X	= 199.892	58.703
Y	= 139.056	– 80.572

Alignment.

$$d = \ell \sin(90^\circ - \theta)$$
$$\Delta = \ell \cos(90^\circ - \theta)$$

d = Distance along alignment line of unknown point
Δ = Distance perpendicular to alignment line of unknown point (misalignment).

X_1, Y_1 = Position of one end of alignment line
X_2, Y_2 = Position of the other end of the line
X_3, Y_3 = Position of unknown point

$$\theta = \left(\tan^{-1} \frac{Y_2 - Y_1}{X_2 - X_1}\right) - \left(\tan^{-1} \frac{Y_3 - Y_1}{X_3 - X_1}\right)$$

$$\ell = \sqrt{(X_3 - X_1)^2 + (Y_3 - Y_1)^2}$$

EXAMPLE

Endpoints	Unknown Position	Alignment
X_1 = 3.016	X_3 = 15.045	Δ = 0.302
Y_1 = 2.989	Y_3 = 14.682	d = 16.773
X_2 = 21.885		
Y_2 = 22.003		

APPENDIX B. REFRACTIVE INDEX CORRECTIONS FOR DME

For precise work, refractive index corrections must be made to distances obtained with distance measuring equipment (DME). Although some instruments have provision for making these corrections in the field by means of nomographs, tables, or slide rules and a dial in parts-per-million correction, it is often better to apply the corrections at a later time. For these cases, the DME parts-per-million correction dial should be set at zero and the following equations used.

First, determine the group refractive index, n_g, for the type of light source used in the DME.

1. Group index of modulated light of wavelength λ at 0° C and 760 mm H_g.

$$n_g = 1 + \left(287.604 + \frac{4.8864}{\lambda^2} + \frac{0.068}{\lambda^4} \right) \cdot 10^{-6}$$

n_g = Group index of refraction at standard conditions
λ = Wavelength of light used in micrometers

Some representative values used in DME are

Wavelength (micrometers)	Source	Refractive Index (n_g)
0.48	Xenon arc	1.0003101
0.63	HeNe (red) laser	1.0003002
0.84	GaAs laser	1.0002947
0.91	GaAs IR diode	1.0002936

2. From the slide rule, nomogram, or tables, or from the manufacturer determine under what conditions of temperature and pressure the instrument requires no correction. Because many combinations of temperature and pressure will provide a zero parts-per-million correction, more accuracy will be obtained by performing the calculations several times using different combinations and taking a mean. From these combinations, calculate the design refractive index n_d.

$$n_d = 1 + \left(\frac{n_g - 1}{1 + \frac{T}{273.2}} \right) \frac{P}{760}$$

where

T is the temperature in C°
P is the pressure in mm of mercury (mm H_g)
n_g is the group refractive index for the light source in the DME

3. Determine the refractive index along the measured line, n_a, from temperature and pressure measurements made at the time. Use the same equation as Step 2.

$$n_a = 1 + \left(\frac{n_g - 1}{1 + \frac{T}{273.2}} \right) \left(\frac{P}{760} \right)$$

where

T and P are now the temperature and pressure measured along the line.

4. Then calculate the corrected distance, D_{corr}, from the observed distance D_{obs}, by

$$D_{corr} = \frac{n_d}{n_a} \times D_{obs}$$

EXAMPLE

A DME uses a laser with a wavelength λ = 0.6328 micrometers
n_g = 1.00030023

The instrument has a 0 ppm correction when T = –8° and P = 760 mm H_g
n_d = 1.00030929

An observed distance of 1208.280 meters is measured on a day when the temperature is 17.2°C and the pressure is 719 mm H_g
n_a = 1.00026753

The correct distance is 1208.330 meters.

APPENDIX C. ATMOSPHERIC CORRECTION TO OBSERVED RATIOS

It was noted in the text that while a ratio of observed distances is very constant with time, it will not equal a ratio of corrected distances unless the midpoint elevations of the two lines forming the ratio are the same (± 25 meters). If the midpoint elevations are different, the observed ratio may be modified by applying a correction factor to each observed distance (D_{obs}) before taking the ratio. The correction factor is a function of the midpoint elevation of the line. After applying the correction factor to each line, the ratio of the two will equal the corrected ratio.

Midpoint Elevation (meters)	Correction Factor X
0	1.0000000
50	1.0000013
100	1.0000026
150	1.0000039
200	1.0000052
250	1.0000064
300	1.0000077
350	1.0000090
400	1.0000103
450	1.0000115
500	1.0000128
550	1.0000140
600	1.0000153
650	1.0000165
700	1.0000178
750	1.0000190
800	1.0000202
850	1.0000214
900	1.0000227
950	1.0000239
1000	1.0000251
1050	1.0000263
1100	1.0000275
1150	1.0000287
1200	1.0000299
1250	1.0000311
1300	1.0000323
1350	1.0000335

Midpoint Elevation (meters)	Correction Factor X
1400	1.0000346
1450	1.0000358
1500	1.0000370
1550	1.0000381
1600	1.0000393
1650	1.0000405
1700	1.0000416
1750	1.0000427
1800	1.0000439
1850	1.0000450
1900	1.0000462
1950	1.0000473
2000	1.0000484

EXAMPLE

Lines AB and AC are the same length, 1206.533 meters.
The ratio of AB/BC is 1.0000000.

D_{obs} for AB is 1206.418 meters
D_{obs} for AC is 1206.411 meters

The observed ratio is 1.0000058
The midpoint elevation of AB is 126 meters.
The midpoint elevation of AC is 357 meters.

From the correction table,

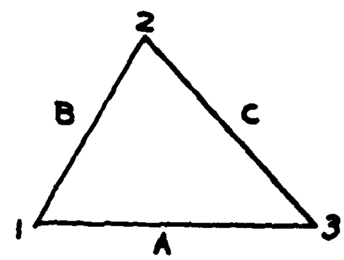
AB = 1206.418 X 1.0000033 = 1206.422 meters
AC = 1206.411 X 1.0000092 = 1206.422 meters

The ratio AB/AC = 1.0000000

Note that both of these lengths are not the true lengths,
but the ratio is the true ratio.

APPENDIX D. ADJUSTMENTS

ADJUSTMENT OF THE THREE RATIOS OF A TRIANGLE



Refer to the figure, and let

$$\frac{A_1}{B_1} = E \qquad \frac{B_2}{C_2} = F \qquad \frac{C_3}{A_3} = G$$

then

$$W = (E \times F \times G) - 1$$

$$E_1 = \frac{E}{1 + \frac{W}{3}} \qquad F_1 = \frac{F}{1 + \frac{W}{3}} \qquad G_1 = \frac{G}{1 + \frac{W}{3}}$$

$E,\ F,\ G$ = Unadjusted ratios
E_1, F_1, G_1 = Adjusted ratios

The adjusted ratios may then be used to calculate the angles of the Δ.

EXAMPLE

	Unadjusted	Adjusted
$\frac{A_1}{B_1} = \frac{1198.106}{1631.110}$	= 0.7345342	0.7345334
$\frac{B_2}{C_2} = \frac{1631.099}{1541.141}$	= 1.0583710	1.0583699

	Unadjusted	Adjusted
$\dfrac{C_3}{A_3} = \dfrac{1541.164}{1198.112}$	$= 1.2863272$	1.2863258
$\dfrac{A_1}{B_1} \times \dfrac{B_2}{C_2} \times \dfrac{C_3}{A_3}$	$= 1.0000032$	1.0000000
	$63^\circ 52' 42''.02$	$63^\circ 52' 41''.94$
	$71^\circ 51' 16''.39$	$71^\circ 51' 16''.45$
	$44^\circ 16' \ 1''.59$	$44^\circ 16' \ 1''.61$

THREE-LINE TRILATERATION ADJUSTMENT

In figure D1, an unknown point U has been measured from three known control points C_1, C_2, and C_3 by means of distance measuring instruments, producing lengths L_1, L_2, and L_3. Three solutions are obtained:

1. U_{12} from C_1 and C_2, L_1 and L_2.
2. U_{13} from C_1 and C_3, L_1 and L_3.
3. U_{23} from C_2 and C_3, L_2 and L_3.

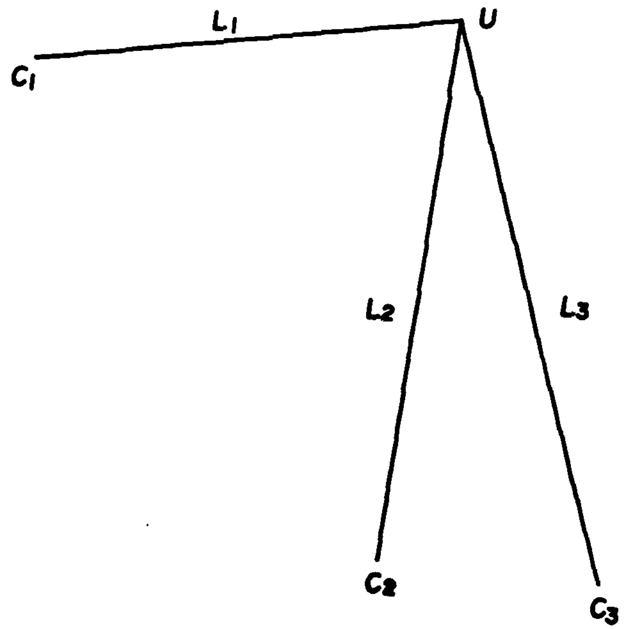


Figure D1. Three-Line Trilateration.

Three new lengths are next computed using the three solutions for U.

1. L_{12} is the distance from U_{12} to C_3.
2. L_{13} is the distance from U_{13} to C_2.
3. L_{23} is the distance from U_{23} to C_1.

Now three weights are determined, one for each solution.

1. $$W_{12} \text{ for } U_{12} = \frac{1}{(L_3 - L_{12})^2} \qquad (1)$$

2. $$W_{13} \text{ for } U_{13} = \frac{1}{(L_2 - L_{13})^2} \qquad (2)$$

3. $$W_{23} \text{ for } U_{23} = \frac{1}{(L_1 - L_{23})^2} \qquad (3)$$

Finally, a weighted mean is taken of the X, Y coordinates of U_{12}, U_{13}, and U_{23} to obtain a final adjusted position. The property of this position, U_F, is that the sum of the squares of the differences between the measured lengths and the calculated lengths (U_F to C_1, C_2 and C_3) is a minimum.

If it is desired to use an *a priori* determined precision for the original lines, then the weights W_{12}, W_{13}, and W_{23} may be modified. If the precision determined for length L_3 is M_3, then equation (1) may be rewritten as follows.

$$W_{12} = \frac{(M_3)^2}{(L_3 - L_{12})^2} \qquad (4)$$

Similarly,

$$W_{13} = \frac{(M_2)^2}{(L_2 - L_{13})^2} \qquad (5)$$

and

$$W_{23} = \frac{(M_1)^2}{(L_1 - L_{23})^2} \qquad (6)$$

where M_2 and M_1 are the precisions determined for lengths L_2 and L_1 respectively.

As an example, the following data is given for the figure D1:

	X	Y
C_1	1125.000	5600.000
C_2	4000.000	1100.000
C_3	6125.000	875.000

$$L_1 = 3680.386$$
$$L_2 = 4767.555$$
$$L_3 = 5100.142$$

Computations of U yield

	X	Y
U_{12}	4799.9497	5799.9639
U_{13}	4799.9474	5800.0060
U_{23}	4799.8526	5799.9805

Then, from these three solutions new lengths are calculated.

$$L_{12} = 5100.1008$$
$$L_{13} = 4767.5961$$
$$L_{23} = 3680.2899$$

The weights are then calculated from the differences

$$W_{12} = \frac{1}{(5100.142 - 5100.1008)^2} = 589.12$$

$$W_{12} = \frac{1}{(4767.555 - 4767.5961)^2} = 591.99$$

$$W_{23} = \frac{1}{(3680.386 - 3680.2899)^2} = 108.28$$

Finally, the weights are used to determine a weighted mean for the adjusted X and Y coordinates.

X

$(U_{12} \times W_{12})$	4799.9497	x 589.12	=	2827746.37
$(U_{13} \times W_{13})$	4799.9474	x 591.99	=	2841520.86
$(U_{23} \times W_{23})$	4799.8526	x 108.28	=	519728.04
	SUMS	1289.39	=	6188995.27

Weighted mean = 4799.940 = X

Y

$(U_{12} \times W_{12})$	5799.9639	x 589.12	=	3416874.733
$(U_{13} \times W_{13})$	5800.0060	x 591.99	=	3433545.552
$(U_{23} \times W_{23})$	5799.9805	x 108.28	=	628021.899
	SUMS	1289.39	=	7478442.173

Weighted mean x 5799.985 = Y

Using this final adjusted position, the three line lengths may be recalculated, and the differences are determined to be

$$L_1 = 3680.386 - 3680.377 = .009$$
$$L_2 = 4767.555 - 4767.574 = -.019$$
$$L_3 = 5100.142 - 5100.124 = .018$$

The sum of the squares of these differences is a minimum for this position of U.

A value for precision could also have been assigned to the measured lengths. One way to do this would be to determine an error for each length based on the specifications of the instrument used for the measurements. The usual specification of error consists of two parts; (1) an error assigned to each measurement regardless of length, and (2) a second error, independent of the first, that is proportional to the length of the line.

The instrument used to measure L_1, L_2, and L_3 has an error specification of 5 mm + 1 ppm of the line length. From this specification, the relative error to be expected from a measurement of L_1 is

$$M_1 = \sqrt{(0.005)^2 + (3680.386 \times 10^{-6})^2} = 0.0062$$

Similarly, for $L_2, M_2 = 0.0069$; and for $L_3, M_3 = 0.0071$.

Substituting these values into equations (4), (5), and (6) gives

$$W_{12} = \frac{(0.0071)^2}{(5100.142 - 5100.1008)^2} = 0.0296977$$

$$W_{13} = \frac{(0.0069)^2}{(4767.555 - 4767.5961)^2} = 0.0281848$$

$$U_{23} = \frac{(0.0062)^2}{(3680.386 - 3680.2899)^2} = 0.0041623$$

The weighted mean may then be calculated as before to obtain a position of

$$X = 4799.942$$
$$Y = 5799.984$$

QUADRALATERAL ADJUSTMENT.

One of the most commonly used trilateration figures is the doubly braced quadralateral. Only five of the six lengths (figure D3) are required to determine the shape and size of the figure. The sixth length provides the required redundancy to check or adjust the figure.

The following method by B. T. Murphy and G. J. Thornton–Smith[1] will adjust the six lengths of a doubly braced quadralateral.

In figure D3, the sides r - - - - - - -w are given. Using the appropriate sides the angles A_1, A_3, B_2, B_3, C_1, C_2, C_3, D_1, and D_2 are calculated using the consine formula given in figure D2. Then the adjusted sides R- - - - - - -W are given by:

$$
\begin{aligned}
R &= k \cdot C_r + r \\
S &= k \cdot C_s + s \\
T &= k \cdot C_t + t \\
U &= k \cdot C_u + u \\
V &= k \cdot C_v + v \\
W &= k \cdot C_w + w
\end{aligned}
$$

where

$$C_r = \frac{\cot D_2 - \cot D_1}{r}$$

$$C_s = \frac{\cot B_2 - \cot B_3}{s}$$

$$C_t = \frac{-\csc B_2}{s}$$

$$C_u = \frac{-\csc D_2}{r}$$

$$C_v = \frac{\csc B_3}{s}$$

$$C_w = \frac{\cot A_3 + \cot A_1}{w}$$

[1]B. T. Murphy, and G. J. Thornton-Smith, *Empire Survey Review*, 1957, Vol XIV, pp175-184.

$$k = \frac{(C_1 + C_3 - C_2) \text{ in arc seconds*}}{(C_r^2 + C_s^2 + C_t^2 + C_u^2 + C_v^2 + C_w^2) \cdot 206265}$$

EXAMPLE

$$\begin{aligned} r &= 6582.423 \\ s &= 4467.924 \\ t &= 6984.762 \\ u &= 9427.710 \\ v &= 10201{:}734 \\ w &= 8358.777 \end{aligned}$$

	Angle	CSC	cot
A_1	42° 55′ 1″.95	————	1.07548022
A_3	32° 18′ 19″.64	————	1.58151016
B_2	91° 1′ 41″.21	1.00016102	−0.01794590
B_3	27° 42′ 19″.80	2.15087514	1.90427515
C_1	56° 39′ 59″.16	————	————
C_2	133° 53′ 55″.95	————	————
C_3	77° 14′ 2″.84	————	————
D_1	18° 23′ 44″.25	————	3.00687746
D_2	59° 50′ 55″.20	1.15646811	0.58087734

$$C_r = \frac{0.58087734 - 3.00687746}{6582.423} = -3.6856\text{x}10^{-4}$$

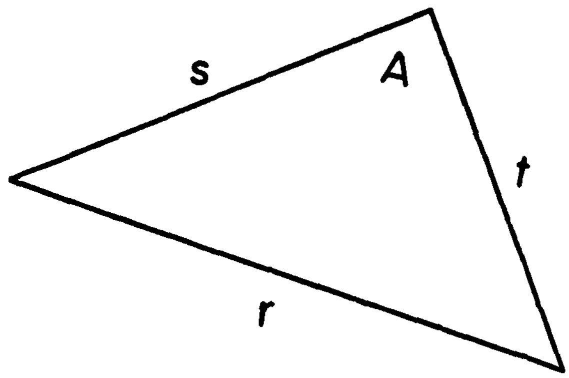
$$C_s = \frac{-0.01794590 - 1.90427515}{4467.924} = -4.3023\text{x}10^{-4}$$

$$C_t = \frac{-1.00016102}{4467.924} = -2.2385\text{x}10^{-4}$$

$$C_u = \frac{-1.15646811}{6582.423} = -1.7569\text{x}10^{-4}$$

$$C_v = \frac{2.15087514}{4467.924} = +4.8240\text{x}10^{-4}$$

*$(C_1 + C_3 - C_2)$ is computed in arc seconds and should be less than 30″. The number 206265 in the denominator is the number of arc seconds in one radian, $\frac{360\text{x}60\text{x}60.}{2\pi}$



$$\text{Cos } A = \frac{s^2 + t^2 - r^2}{2\,st}$$

Figure D2. Cosine Law and Triangle.

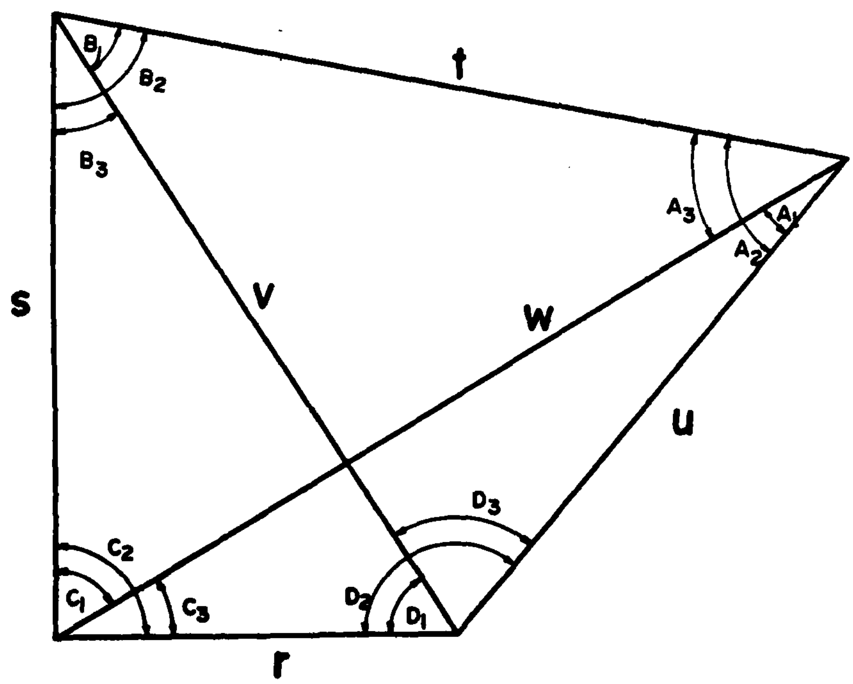


Figure D3. Doubly Braced Quadralateral.

$$C_w = \frac{1.58151016 + 1.07548022}{8358.777} = +3.1787 \times 10^{-4}$$

$$k(\text{numerator}) = 56^\circ\ 39'\ 59''.16 + 77^\circ\ 14'\ 02''.84 - 133^\circ\ 53'\ 55''.95 = 06''.05$$

$$k(\text{denominator}) = \left[(-3.6856)^2 + (-4.3023)^2 + (-2.2385)^2 + (-1.7569)^2 + (4.8140)^2 + (3.1787)^2\right] \times 206265 \times 10^{-8} = (73.4697 \times 206265 \times 10^{-8} = 0.15154$$

$$k = \frac{6.05}{0.15154} = 39.92$$

LENGTH CORRECTIONS

$$kC_r = 39.92\ (-3.6856 \times 10^{-4}) = -0.0147$$
$$kC_s = 39.92\ (-4.3023 \times 10^{-4}) = -0.0172$$
$$kC_t = 39.92\ (-2.2385 \times 10^{-4}) = -0.0089$$
$$kC_u = 39.92\ (-1.7569 \times 10^{-4}) = -0.0070$$
$$kC_v = 39.92\ (+4.8140 \times 10^{-4}) = +0.0192$$
$$kC_w = 39.92\ (+3.1787 \times 10^{-4}) = +0.0127$$

ADJUSTED LENGTHS

R	=	6582.423	–	0.0147	=	6582.408
S	=	4467.924	–	0.0172	=	4467.907
T	=	6984.762	–	0.0089	=	6984.753
U	=	9427.710	–	0.0070	=	9427.703
V	=	10201.734	+	0.0192	=	10201.753
W	=	8358.777	+	0.0127	=	8358.790

APPENDIX E. ERROR ESTIMATION FOR SIMPLE CASES OF TRILATERATION

In the measurement of movements of dams or other large structures using trilateration, it is important to locate control monuments in the proper places. Topographic and other restrictions often eliminate the best locations, leaving the surveyor with several, less than ideal, possibilities. The purpose of this appendix is to provide the surveyor with a means of estimating the accuracy, in two dimensions, of the position of a point when length measurements are made to it from two known positions.

Figure E1 shows two control monuments, A and B, from which lengths have been measured to an unknown position C.

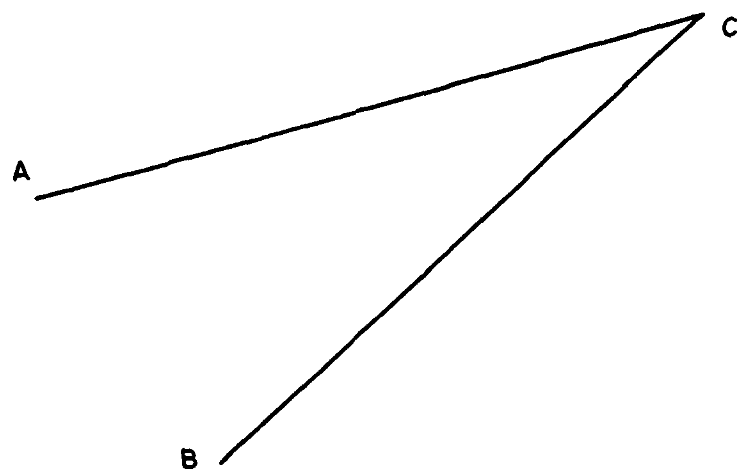


Figure E1. Point C Measured from A and B.

Figure E2 is an enlarged view of the area around C. If a large number of measurements, containing random errors, were made of C from the two control points, they would be found to lie in an ellipse around the true position of C. The dimensions of the ellipse may be estimated from the relative geometry of the three points and the error specification of the instrument.

The error specification of the instrument is generally written in the form (a) mm ± (b) ppm. The (a) portion is a fixed error of (a) mm. The (b) portion is proportional to the length of the line. An example would be 5 mm ± 3 ppm. For a 2,000-meter line, the (a) portion of the error would be 5 mm and the (b) portion would be $2000 \times 3 \times 10^{-6}$ or 6 millimeters. The errors must then be added statistically to obtain the total error $= \sqrt{a^2 + b^2}$ or in the example $\sqrt{5^2 + 6^2} = 7.8$ mm.

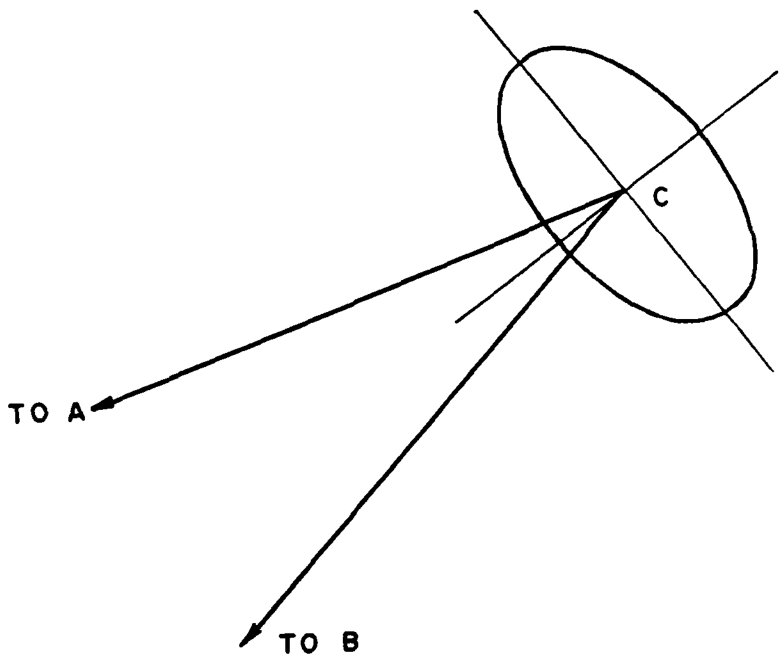


Figure E2. Enlarged View at C.

In figure E3, the error ellipse is estimated by means of a paralelogram. The size, shape, and orientation of the parallelogram are obtained from the positions of the control monuments, the lengths of lines, and the instrument errors for each line. From a sketch or map of the locations of A, B, and C, it is possible to assign approximate coordinates to the three positions and from these to determine the lengths L_a and L_b. If X_A, Y_A and X_C, Y_C are the coordinates of A and C, then $L_a = \sqrt{(X_A - X_C)^2 + (Y_A - Y_C)^2}$.

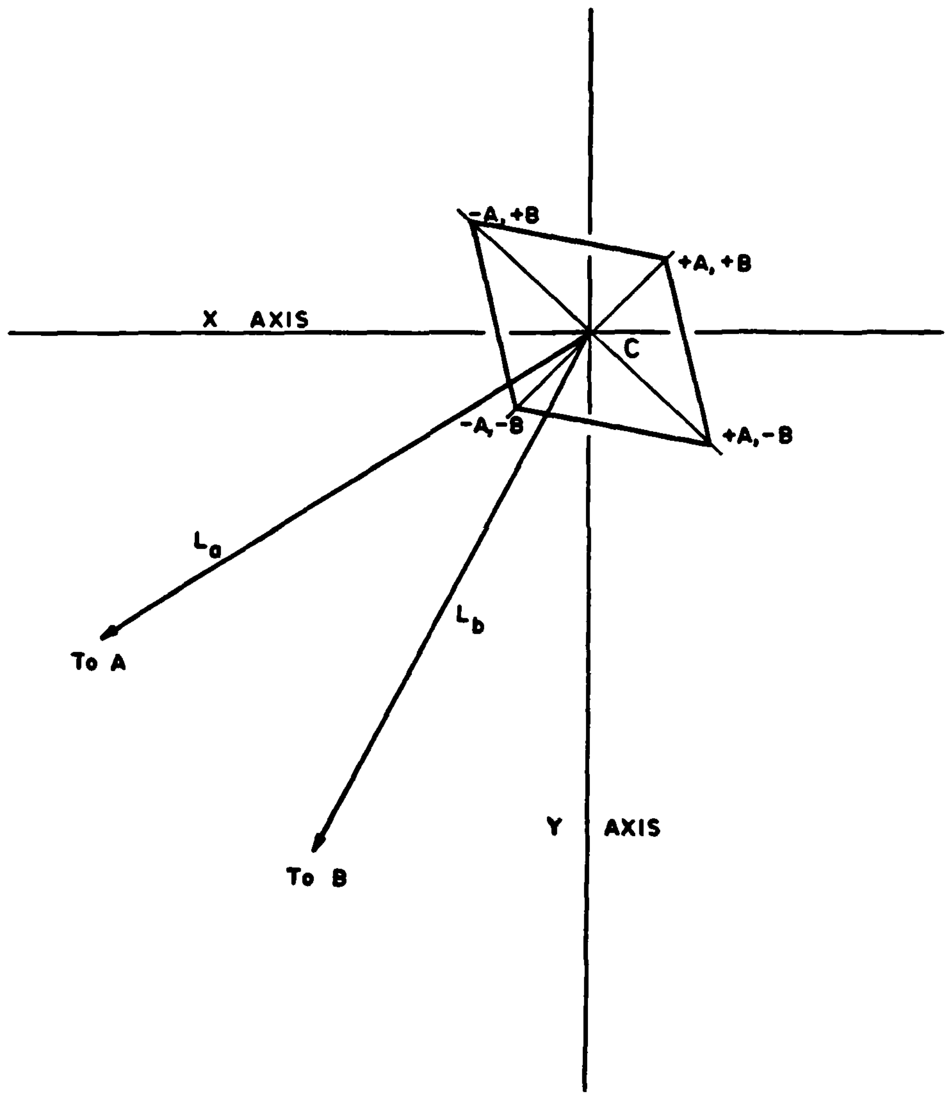


Figure E3. Error Parallelogram.

When the lengths L_a and L_b have been calculated, they may be used together with the instrument error to obtain the four corners of the parallelogram.

In figure E3, the corner +A, +B, is obtained when the instrument error for line L_a is added to L_a and when the instrument error for line L_b is added to L_b. Then these two new lengths are used together with the coordinates of A and B. By using the errors in combination, the other three corners of the parallelogram are then found. In practice, it is necessary to determine the coordinates of only two of the corners, for example −A,+B and +A,+B, because the other two corners are symmetrical about the point C. Finally, if the X and Y coordinates of the parallelogram corners are subtrated from the coordinates of position C, an estimate of the error at C from A and B is obtained.

EXAMPLE

Results are easier to understand if the X or Y axis of the map or sketch is parallel with the dam axis. In figure E4, the sketch has been oriented so that the dam is parallel with the X axis. Points A, B, and C have been scaled from the map.

	X	Y
A	300	600
B	1200	500
C	1900	2000

From these, the lengths of lines L_a and L_b have been calculated as

$$L_a = 2126.029 \text{ meters}$$
$$L_b = 1655.295 \text{ meters}$$

The instrument error is 5 mm ± 3 ppm. For line L_a, the error would be 8.1 mm and for line, L_b 7.0 mm.

$$L_a \text{ error} = \sqrt{(2126 \text{x} 3 \text{x} 10^{-6})^2 + 5^2} = 8.1 \text{ mm}$$
$$L_b \text{ error} = \sqrt{(1655 \text{x} 3 \text{x} 10^{-6})^2 + 5^2} = 7.0 \text{ mm}$$

Next, a table of line lengths may be made as follows:

	+ error	– error
Line L_a	2126.0371	2126.0109
Line L_b	1655.3020	1655.2880

Now, two new positions of C (the corners of the error parallelogram) are calculated using the positions of A and B and, first, lengths 2126.0209 and 1655.3020 and, second, lengths 2126.0371 and 1655.3020. From the first two, the coordinates

$$X = 1899.9693$$
$$Y = 2000.0226$$

are obtained, and from the second set, the coordinates are

$$X = 1900.0056$$
$$Y = 2000.0056$$

Finally, the errors are calculated

$$X = 1900 - 1899.9693 = +30.7 \text{ mm}$$
$$Y = 2000 - 2000.0226 = -22.6 \text{ mm}$$

and

$$X = 1900 - 1900.0056 = -5.6 \text{ mm}$$
$$Y = 2000 - 2000.0056 = -5.6 \text{ mm}$$

The maximum error is the one of interest, 30.7 mm in the X direction and 22.6 mm in the Y direction. Depending on the orientation of the lines, the maximum X or Y may appear in either calculation so that it is necessary to perform both sets of calculation and choose the large set.

In this case, the errors are large because of a poor selection for A and B. If the lines L_a and L_b intersected at 90° at C, much better results would be obtained, approximately 7 mm in both X and Y. Figure E4 shows the shape and orientation of the error parallelogram for the example. Note how rotation of the parallelogram (or the coordinate system) does not change its shape, but influences the maximum X and Y error.

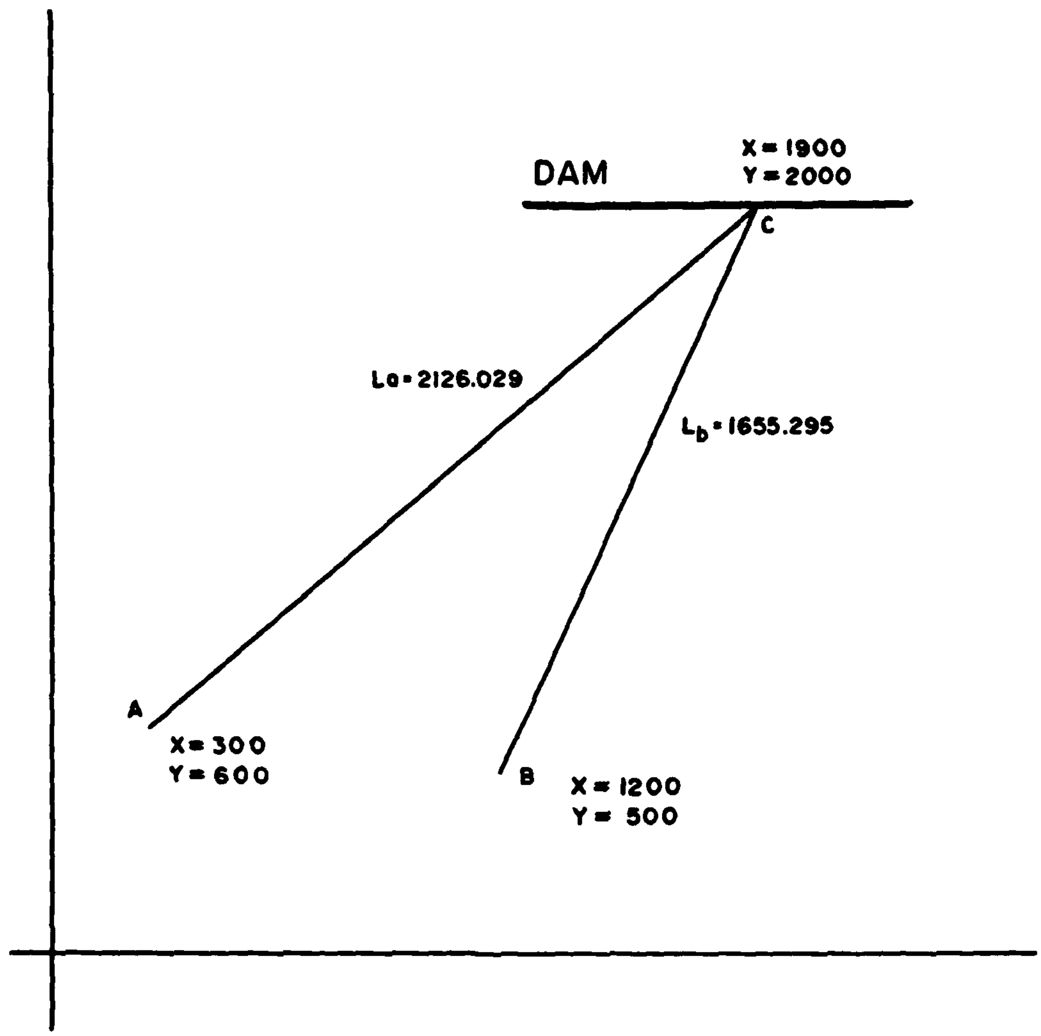


Figure E4. Example.

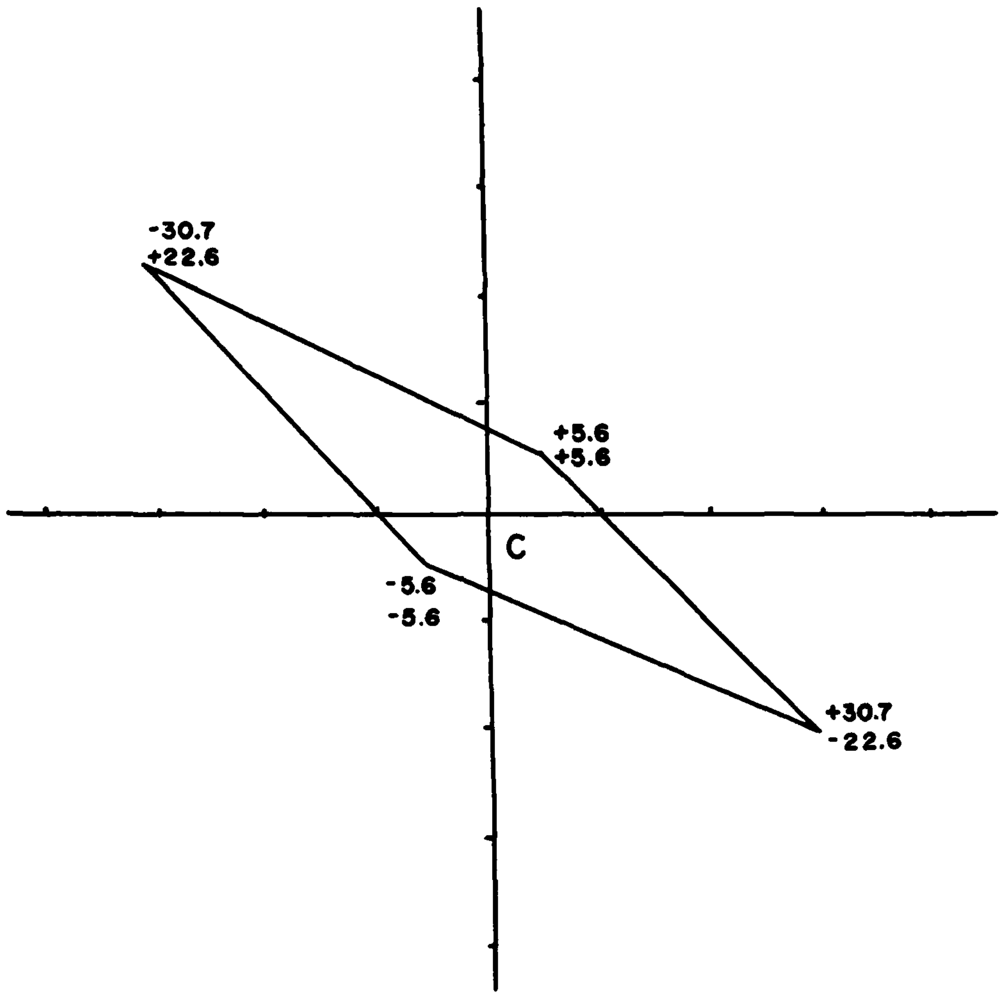


Figure E5. Example Parallelogram.

1 3 4 88,096—AG—Ft Belvoir